Bipedal Locomotion Up Sandy Slopes: Systematic Experiments Using Zero Moment Point Methods

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Abstract—Bipedal robotic locomotion in granular media presents a unique set of challenges at the intersection of granular physics and robotic locomotion. In this paper, we perform a systematic experimental study in which bipedal robotic gaits for traversing a sandy slope are empirically designed using Zero Moment Point (ZMP) methods. We are able to implement gaits that allow our 7 degree-of-freedom planar walking robot to ascend slopes with inclines up to 10°. Firstly, we identify a given set of kinematic parameters that meet the ZMP stability criterion for uphill walking at a given angle. We then find that further relating the step lengths and center of mass heights to specific slope angles through an interpolated fit allows for significantly improved success rates when ascending a sandy slope. Our results provide increased insight into the design, sensitivity and robustness of gaits on granular material, and the kinematic changes necessary for stable locomotion on complex media.

I. INTRODUCTION

Enabling humanoid machines to walk wherever humans can go is an ongoing robotics challenge. This challenge is especially apparent when walking on the complex and dynamic terrain present in the natural world, such as a sand dune – a sloped surface of granular material. Every footfall will sink into the non-rigid terrain exhibiting variable dynamics with respect to time and space. Additionally, an intruding foot can cause the loose material to yield, thereby slipping backward and deviating significantly from the planned walking dynamics. Further, the sand grains themselves may avalanche down the slope under pressure, creating unanticipated changes in the ground reaction forces. All of these factors can contribute to robot falls. In this paper, we experimentally investigate the control problem of robotic walking up a sandy slope by systematically adjusting the free design parameters present in established Zero Moment Point (ZMP) techniques [1, 2]. In the past, many walking robots have made use of control methods such as Hybrid Zero Dynamics (HZD) [3, 4], Zero Moment Point (ZMP) [1], and other methods of keeping the body upright [5], to ensure quasistatic or dynamic stability of the system. These methods can be then used to generate predetermined planar, or even three-dimensional paths for the biped system to follow [6]. Many of these schemes have been implemented across a variety of environments, and are typically augmented by position and acceleration feedback [1, 3, 7–15]. Methods which further employ force-based feedback can accommodate deformable terrain [16–21]. However, granular terrain poses a particular challenge given its highly nonlinear dynamics and ability to both act like a solid and flow in a fluid-like regime (yielding or avalanching) during foot contact [22, 23]. As a consequence,
if the terrain transitions from a solid-like material to flowing material as a result of the foot fall [24, 25], then traditional force control for gait recovery may not accommodate such a drastic dynamical shift in the interaction of the foot surface and the ground.

This paper builds on previous work using a 7 degree of freedom planar walking robot [2], in which we developed a stability region criterion for generating ZMP-based flat-footed walking on granular terrain, and applies the approach to the additional complexity of sloped granular terrain (Fig. 2, Section III). This work systematically varies gait parameters and experimentally assesses which adjustments yield effective gaits for planar locomotion on sandy slopes (Section IV). We generalize these effective gait parameters into a slope-dependent fitted function (in essence, a parameter lookup table) which results in ascending our experimental granular slope, shown in Fig. 1. Further, we perform a sensitivity study to show these generated gaits are robust to misestimation of the slope angle (Section IVC). For future work, this array of generated gaits can provide a baseline open loop scheme for higher-level feedback from inertial measurement units and force-sensing in the feet (Section VI).

II. BACKGROUND

A. Balancing of Bipedal Walking

Perhaps one of the most intuitive and widely implemented methods to balance a biped robotic system, and account for environmental disturbances, is through position and acceleration feedback, achieved by installing inertial measurement units (IMU’s) at strategic points on the head or body of the system. Much like human reflexes, this sensory data is fed into a control center that then signals motion or repositioning of the torso or appendages as necessary. However [9], the idea of using this information to modify our open loop gait kinematics on granular material by taking advantage of the material properties of the substrate itself, has yet to be explored. This provides context for the experiments presented in this paper.

Typical bipedal walking has an alternating single and double support phase (SSP and DSP) [2], which is well studied for ground where a robot’s feet do not have complex interactions, such as penetration or slip on the boundary of the surface. For biological systems, these complex surfaces are typically accounted for through sensing and processing of the resulting forces on the plantar surface of the foot, using input as a feed-forward into a complex, proprioceptive feedback system. In fact, it is difficult to enable and optimize walking in a robotic system over sand without estimation of the complex force interactions that occur at the surface [22, 23]; a phenomenon which was built upon in our previous work [2]. However, one of the key biological corrections that bipeds make using this force feedback is that of kinematic repositioning [26]. This supports the idea that adjusting the gait kinematics can allow slope climbing on complex terrain for biped robots. This was previously investigated for simulated human gait through variation of the parameters of step height, center of mass (CoM) height and step length [10]. These used simulated vestibular IMU input to control torso position, as well as a simulated kinesthetic sense through force and torque sensors to govern joint positions. The focus was a kinesthetic augmentation of the vestibular sense to provide stability when walking over complex terrain, and served as a good indicator of gait adjustments for uphill gait design.

B. Balancing of Bipedal Walking on Staircases

in order to traverse a slope or inclined surface, the regular ZMP equation must be modified. This is because of the need to accommodate the fact that in any given phase where both feet make contact with the surface, one leg will be higher than the other [11]. Since traditional ZMP trajectories assume the contact point of both feet must be coplanar [11, 12], we considered the uphill trajectories of the robot to be along a virtual slope.

III. CONTROL AND EXPERIMENTAL METHODS

A. Walking on Flat Surfaces

ZMP Walking is based on the idea of the Linear Inverted Pendulum Model (LIPM) of bipedal walking, built upon the concept of a stability region in which stable gaits can be generated. One main characteristic of ZMP is that at any given moment, the trajectory of the gait places the biped within the stable region of the LIPM. Because the background work for ZMP walking assumes a constant vertical center of mass (CoM) height and constant foot position, for flat ground this position is determined using the following equations [1]:

\[
P_{x-zmp} = \frac{-\tau_y}{mg} \tag{1}
\]

\[
P_{y-zmp} = \frac{\tau_y}{mg} \tag{2}
\]

Where \(\tau_x\) and \(\tau_y\) are the torques on the ankle of the biped walker due to inertial effects of the biped, \(g\) is acceleration.
Our previous work demonstrated that the static placement of the foot allowed for the reactive terrain forces to balance the ankle torques. As such, the estimated stability region of the granular material needed to be equivalent to the stability region of the LIPM ZMP model. It is necessary to design trajectories such that that stability criterion is satisfied for all time [2]. When we factor in the approximate sinkage of the granular matter to be traversed, we arrive at the model to calculate the trajectory of the robot joints \( T_{ZMP} \), which can be given as a function, and is explored further in [2]:

\[
T_{ZMP} = f(\text{CoM}, L, h, S, g, B_{f_{\text{front}}}, B_{b_{\text{back}}})
\]  

(6)

Where \( \text{CoM} \) is the center of mass height, \( L \) is the biped step length, \( h \) is the step height, \( S \) is the sinkage of the biped, and \( B_{f_{\text{front}}} \) and \( B_{b_{\text{back}}} \) are the stability conditions of the biped for its given mass and size constraints.

To quantify our foot size, and thus \( B_{f_{\text{front}}} \) and \( B_{b_{\text{back}}} \) [2], we adapted the idea of the "polygon of stability" for locomotion on hard ground for use on granular media [2]:

\[
d_c = \frac{L}{2}\left(1 - \frac{1}{\gamma}\right)
\]

(7)

where we consider relationship of the planar foot length, \( L \), to intrusion depth \( d_c \) and \( \gamma \) is the force overshoot ratio, which factors in force and velocity of the intrusion into the granular material, and can be obtained from literature [2].

C. Challenges of Walking on a Slope of Granular Terrain

Because the surface of the slope is that of granular material, a gait that addresses uphill climbing must be used with the same accommodations made for sinkage and equivalence of the stability regions. When designing our approach to this problem, the following were considered:

1) Slippage and uniform compaction: To provide a model of constant sinkage which could be estimated and eventually fed into a ZMP trajectory equation, it is necessary to have predictability of the behavior of the bed. A fluidized bed was built, as shown in Fig. 3, that allows for the repeatable preparation of uniform and loosely packed granular material, in this case poppy seeds [23]. Blowing air up through the base of the bed fluidizes the substrate when turned on, and after deactivating the blowers and letting the grains settle, a loosely packed and uniform granular surface remains. To accurately feed sinkage information into our ZMP trajectory generation algorithm, we first needed to know what the sinkage of the robot would be at a given weight, to address this, a simple measurement of parameters was conducted, as seen in Table 1. If the sinkage is not properly accounted for, we find that the robot gait will not compensate for the extra sink distance, and thus, the gait will fail and the robot will tip over.

2) Failure of the substrate itself: Another key challenge that we face when walking uphill on granular material is the failure of the surface itself, due to the effects

| **TABLE I: Robot and bed sinkage parameters for the 7 degree of freedom planar walker** |
|------------------|----------|
| **Quantity**     | **Value** |
| Robot Mass       | 1.86 kg  |
| Plantar Surface Area | 100 cm³  |
| Intrusion Depth  | 1.8 cm   |
| Foot Length      | 20 cm    |
| Foot Height      | 5 cm     |
of gravity, frictional forces, and lateral forces applied by the surface of the robot foot [24]. These complex interactions have been found to occur in slopes as low as 6° for the robot, and will be further discussed in Section V.

D. Developing a Staircase Gait

Staircase gaits have been studied with respect to ZMP trajectories for many years [11,12]. Similarly, ZMP trajectories over uneven or complex terrain have also been very thoroughly explored, due to the fact that biped robots rarely operate in perfectly ideal situations [2,10]. We seek to combine much of the work that has been done in order to allow for robust uphill walking uphill over granular material.

Because of inconsistencies in the ZMP trajectory formulation during the DSP of the walker, we use a virtual slope model [12]. Thus, we are left with an expression for stair climbing up a virtual slope that was implemented as our ZMP staircase equation [12]:

\[ P_{\text{zmp}} = \frac{k_1 + g}{g} \ y - \frac{z}{g} \]

This equation allows the trajectory of the foot to follow a staircase pattern, while allowing its CoM to ascend the slope in a linear manner. However, implementation of a staircase gait by itself is not sufficient to implement on a slope with sinkage. Two major modifications were made to the trajectory generation to provide a novel, robust, and stable gait. Because the differences in slope angles are trigonometically related to the "height of the stairs", the slope angle and intrusion angle of the foot were necessary augmentations of (θ), such that the function to generate \( T_{\text{ZMP}} \) must include θ and φ as inputs:

\[ T_{\text{ZMP}} = f(CoM, L, H, S, g, B_{\text{front}}, B_{\text{back}}, \theta, \phi) \]

Where θ is the slope angle of the sand, and φ is the intrusion angle of the foot, and φ/2 the offset of the hip joints. Once this gait was developed, it was found that a traditional spline fit [2] of the ZMP trajectory path was sufficient only up to the point in the gait cycle that the robot seeks to compensate for sinkage (at the end of two full steps, or one full gait cycle). Because of this, a Savitsky Golay [15] criteria, or piecewise spline with history, was designed and used to generate a smoothed staircase trajectory for the robot to follow.

TABLE II: Joint trajectory generation for the planar walker

<table>
<thead>
<tr>
<th>Algorithm for Joint Trajectory generation</th>
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<tbody>
<tr>
<td>1) Slope angle read in: L and CoM computed: L. CoM = (θ)</td>
</tr>
<tr>
<td>2) ZMP Trajectory Reference computed: ZMP Ref = (qDSPi-nDSPi, nSSPi; nsteps, Bfront, Bback, L)</td>
</tr>
<tr>
<td>3) ZMP Ref used to calculate CoM trajectory from Preview Control: CoM Traj = (ZMP Ref, H, CoM, T, g, θ)</td>
</tr>
<tr>
<td>4) CoM Traj used to plug into the robot inverse kinematics In general: Q = (CoM Traj, backgoal position, frontgoal position)</td>
</tr>
<tr>
<td>Q DSPi = (CoM Traj, 0, sink+(L/2)tan(θ))</td>
</tr>
<tr>
<td>Q DSPF = (CoM Traj, 0, sink+(L/2)tan(θ))</td>
</tr>
<tr>
<td>Q DSPF = (CoM Traj, sink+(L/2)tan(θ), sink+(L/2)tan(θ))</td>
</tr>
<tr>
<td>5) Repeat for nsteps if nsteps is complete: Q DSPF = (CoM Traj, sink+(L/2)tan(θ), sink+(L/2)tan(θ))</td>
</tr>
<tr>
<td>6) Applying smoothing and adjustments: Q intermediate = (Q DSPi+nstepsQ DSP1+Q DSP2 +Q DSPF)+Q DSPF</td>
</tr>
<tr>
<td>Q final = Savitsky-Golay(Q intermediate, θ)</td>
</tr>
</tbody>
</table>

Fig. 4: Frame capture for the biped walker ascending a 4 degree incline. The robot begins ascending the slope by starting in the initial DSP (a), and continues to the first SSP (b), (c) until it arrives at the first intermediate DSP (d). From here, it switches feet, and the enters into the second SSP (e), (f), before arriving at the second DSP (g). The cycle then repeats (h) with the same SSP from (b), until the robot completes the commanded number of steps. From (a) to (h), the red line indicates the position of the center of mass at a given time. The true center of mass was used instead of the at the hip due to the low mass of the system.
E. Joint Angle Methods

Table 2 outlines the algorithm for the calculation of the joint trajectories. The process of Preview Control [1, 2, 11, 12] was used to generate ZMP references, which are then converted to joint angles through inverse kinematics. The necessary intrusion angle of the foot is represented by $\phi$, and the forward hip posturing can be given as $\phi/2$. $s_{DSP0}$ is the set number of initial double support samples, $s_{DSP}$ is the number of double support samples, $s_{SSP}$ is the number of single support samples, $n_{steps}$ is the number of steps, $Q_{initial}$ is the initial joint position matrix, $Q_{SSP1}$ is the first SSP joint position matrix, $Q_{DSP1}$ is the first DSP joint position matrix, $Q_{SSP2}$ is the second SSP joint position matrix, $Q_{DSP2}$ is the second DSP joint position matrix, $Q_{SSPfinal}$ is the final SSP joint position matrix, $Q_{DSPfinal}$ is the final DSP joint position matrix, $Q_{intermediate}$ is the pre-processed joint position matrix, $Q_{final}$ is the final joint position matrix, and $H$ is the preview horizon. An example of the implementation of this algorithm can be seen in Fig. 4.

IV. EXPERIMENTAL TRIALS AND RESULTS

A. Relation of Intrusion Angle and Sinkage to Slope Angle

We first sought to experimentally determine the ideal angle of foot intrusion for our system. From a systematic experimental sweep of foot angles, we found that the optimal foot angle was:

$$\theta = \phi.$$ (10)

The results of this experiment can be seen in Fig. 5, and was conducted as following:

1) Establishing of Reference Frames: The lab frame was used as reference, with the floor being $0^\circ$. The bed of poppy seeds was elevated to $5^\circ$.

2) Sweep of Angles of Intrusion: For $N=5$ trials for each slope angle, the walker was run for $n=4$ steps, and a success was defined as the walker standing upright on two feet at the end of the trial, else a failure. This gave an experimental data set size of 96 total trials.

We hypothesize that a “placing” instead of a “digging” foot motion was more successful due to the relatively low mass and plantar pressure of the walker, compared to the “digging” behavior of humans which are significantly heavier [26].

B. Step Length, CoM and Angle Parameterization

Initially, the relationship in (8) was considered, and through systematic experimentation, a set of parameters ($CoM$, $L$, $H$, $S$ and $B_{front}$ and $B_{back}$ being taken from our previous work) was identified that allowed for a stable gait at $\theta = 5^\circ$. The gait was then tested for a sweep of slope angles from $\theta = 0$ to $\theta = 10^\circ$ to test the general effectiveness of these parameters. The method used was:

1) Parameters that allowed for successful trials at $\theta=5$ were used to generate a trajectories for $n=4$ steps. A success was defined as the walker completing all 4 steps and returning to an initial DSP position. The bed was leveled and the ground was re-fluidized and reset before each trial.

2) A sweep of trials from $\theta=0$ to $\theta=10^\circ$ with the modified staircase code was then run, with $\phi=\theta$ and $n=10$ trials. This gave an experimental control data set size of 100 total trials.

3) Failure modes were analyzed and processed. After this, through systematically incrementing and decrementing our values at each angle, a set of parameters $CoM$, $L$, $H$ were also established for $\theta=0^\circ$ to $\theta=10^\circ$, with a success being defined as the largest values of $L$ and $CoM$ to allow for at least 50% success for $N=10$ gait trials at a given $\theta$. This metric of success was chosen.

Fig. 6: Manually tuned gaits for specific slopes. These were then used to formulate a parameterized policy through the use of a linear fit. In (a), we see the results for Vertical CoM height, and in (b) we see the same experiment for step length. In order to accommodate increasing slope angles, we found that decreasing the Step Length and CoM height was necessary.
due to the fact that we are primarily concerned with the locomotion of the robot on a granular incline, and as such, a larger \( L \) and \( \text{CoM} \) were favored.

4) The successful parameters from \( \theta = 0^\circ \) to \( \theta = 10^\circ \) were then fit with a linear regression, and equations that related all parameters to slope angle were identified for the planar control. (Fig. 6).

5) Step 2) was then re-run, this time using the linear fit to select parameters. This gave an experimentally manipulated data set size of 100 total trials.

At this point, because the step length, \( \text{CoM} \) and step height were all related through a system of continuous linear equations, the robot had a specific gait for each slope angle within the limits of the hardware setup (angles of \( 0 - 10^\circ \), with a resolution of \( 0.01^\circ \)). Once this parametrization was applied, we found that the robot was significantly more successful at traversing the sandy slope, due to the specificity of each gait to a particular incline. The results of this can be seen in Fig. 7. These results are encouraging, since we were able to improve our success rates at \( \theta = 9^\circ \) and \( \theta = 10^\circ \) from 0% to 60% and 50% respectively; a substantial improvement over our initial trial.

C. Sensitivity Trials for Slope Angle

Once the gait was parameterized to be slope-specific, we tested the robustness of the parameterized gaits of the walker [27]. This served as a practical test of our method to mis-estimations of the slope angle. In order to do this, a second experiment was designed:

1) A slope angle of \( \theta = 2^\circ \) was input into the gait code, for \( n = 6 \) steps. A success was defined as the walker completing all 6 steps and returning to an initial DSP position. The bed was leveled and the granular terrain was reset through the air fluidized before each trial.

2) The bed was raised to the \( \theta \) input into the code, and the robot was run \( N = 3 \) times, each time recording the distance, and at the end of the third trial, establishing an average distance.

3) The bed was lowered to an angle of \( (\theta-1^\circ) \), while keeping an input angle of \( \theta \) in the code, and the robot was run \( N = 3 \) times, each time recording the distance, and at the end of the third trial, establishing an average distance. This distance was then divided by the average distance at \( \theta \) to obtain a relative distance traveled. This is important due to failure and slippage of the robot.

4) The bed was again lowered to \( (\theta-2^\circ) \) and the robot was run \( N = 3 \) times, each time recording the distance, and at the end of the third trial, establishing an average distance. This distance was once again divided by the distance traveled at slope \( \theta \) in order to obtain a relative distance traveled. This process was further repeated at bed angles of \( (\theta+1^\circ) \) and \( (\theta+2^\circ) \).

5) The entire process was then repeated for \( \theta=4^\circ, 6^\circ, 8^\circ \) and \( 10^\circ \), giving a total of 150 experimental trials.

For low angles, \( (\theta<6^\circ) \) we find that there is a reasonable level of robustness to perturbation in the open loop gait (Fig. 8). This is indicated by the high relative distances(fraction of distance covered at a particular offset angle, compared to the distance traveled when the slope angle is accurate to the gait) at lower inclines. However, at higher granular inclines, we observe large drop-off on either side of the sensitivity curve, once the gait is not specifically designed for that particular incline. It is this key fact that frames our current and future work.

V. DISCUSSION

A. Theoretical Basis For Experimental Success

As seen in 7 and 8, through the use of a simple linear fit, we were able to increase success rates from from 0% at \( \theta = 9 \) and \( \theta = 10 \) to 60% and 50% respectively. We theorize that this is due to the significant re-posturing of the 7-DOF system according to slope angle to remain within the polygon of stability. Similarly, this explains why the sensitivities drop off quite sharply at higher angles. If we consider that the support length normal to the lab frame is given by \( L_{\cos(\theta)} \) (5), we find that the polygon of stability becomes smaller as \( \theta \) increases, thereby leading to more sensitive gaits.

B. Improving Sensitivity

As we examine the sensitivity curves, it becomes evident that the given scheme of gait generation is effective when the walker receives an accurate slope angle as input. However, for smaller perturbations, modeled as small offsets in slope angle from expected angle, we see that the gait effectiveness falls significantly – as much as 100% for 2 degree perturbations. Because of the nonlinear properties of granular material [24], we need to understand the force profiles at the foot-material boundary [23]. Unlike conventional balance control, to optimize biped walking over granular material it is necessary to characterize how the substrate itself actually yields. Once this is done, we hypothesize that through simple inertial control, for instance, in-the-loop torso repositioning, we can begin to improve sensitivity by combining the IMU...
and force feedback as a more biomimetic approach to biped walking on complex media.

C. Future Work

to expand upon the idea of using force profiles, future work will focus on using force sensing in the feet to inform real-time feedback control for the biped. We have installed both heel and toe force-moment sensors with the goal of detecting, in real time, if the gait is not progressing as planned. This force-based method of feedback also has the added advantage of distinguishing not only whether the machine is falling, but also the cause of the failure on complex terrain. Specifically, force feedback could be used to determine if the foot is slipping, avalanching the material, or hitting an unexpected bump, and then the robot adjust the joint kinematics accordingly in order to recover.

Another aspect that we are currently exploring is that the actual commanded distance does not always equate to traveled distance on a slope of granular material (Fig. 9). After inspection of videos of the foot-substrate interaction and flow, we hypothesize that this is due to slippage during the DSP, in which the robot is stable, but still slides incrementally backward as it positions its second foot. This is seen in Fig. 10.

VI. CONCLUSION

Through this study, we have examined the process of design, implementation and testing of a gait generation approach in which biped robots can be made to walk up a sandy slope. We introduced a robotic walking system and tested it using systematically determined parameters that successfully walked for a certain slope angle. We then related the kinematic parameters, particularly step length and CoM height, as an interpolated function of the slope angle itself, in order to traverse an uphill path of granular terrain. We found that the biped robot had the most success with the interpolated gait parameters being used as a result of sensing the slope angle. We further observed that as the slope intensity increases, the robustness of these gaits to perturbations significantly decreases. We hope to build upon these findings to improve the gait sensitivity through the use

Fig. 9: Graph showing the actual and commanded distance for slope angles between 2° and 10°. This allows us to better understand and characterize the slipping (backward motion of the robot foot due to sliding on the granular surface) of the robot for each slope angle. As can be seen, the robot slips more as the slope becomes steeper
of real-time feedback control and plantar force sensing.

APPENDIX

Supplementary footage for this experimental study may be found on the CRAB Lab Biped Walking Channel.

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