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Once-baffling success of granular resistive force theory explained

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thought. Two groups of theorists, one led by Serge Fehr and the other by Anthony Leverrier, soon proved that the cheating probability doesn't scale double-exponentially with m, but linearly.[2] With that realization, longer bit-commitment times immediately became possible, with reasonable security over not just 6 rounds, but billions. Martin, Zbinden, and colleagues now had the luxury of using smaller random numbers (strings of 128 bits rather than 512) and shorter distances between agents, all within the city of Geneva.

The experimenters placed the computers representing Alice and Bob in the same room and connected them with a 1 m optical link. Just 23 light-microseconds away, Amy and Brian had a similar setup. With so little separation between the pairs, timing was critical to satisfying the relativistic constraints. To account for the nonzero duration of each round and to allow a small tolerance for error, the researchers chose to begin each round of the protocol 17 μs after the previous one. Each pair was synchronized to a GPS receiver that kept time with 150 ns accuracy.

All told, the 24-hour commitment of a single bit required 162 GB worth of random numbers; a year-long commitment would require 59 TB. That amount could be reduced by moving the agents farther apart and lengthening each round of the protocol. For example, if Alice and Bob were 10 000 km away from Amy and Brian, a year-long commitment could be achieved with just 81 GB.

Johanna Miller

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Once-baffling success of granular resistive force theory explained

A model designed to approximate swimming in water accounts surprisingly well for animals' locomotion in sand.

FIGURE 1. THE SANDFISH SKINK, native to the deserts of North Africa and southwestern Asia, eludes predators by diving below the sand surface and swimming.

How is a Mars rover like a sandfish skink? Whether you're a $2.5 billion robot carefully rolling across Martian soil or a 10-cm-long North African lizard (see figure 1) that burrows in the desert sand to evade predators, your locomotion obeys the same set of rules that govern the deformation of granular media.

For close to a decade, researchers have successfully applied an empirical scheme called resistive force theory (RFT) to describe locomotion in dry granular environments. First proposed in the 1950s to calculate speeds of sea-urchin spermatozoa swimming in seawater,[1] RFT approximates locomotion in viscous fluids relatively well, but it's far from perfect. For granular media, though, it works bafflingly well.[2]

To figure out the secret behind granular RFT's success, Ken Kamrin of MIT and his postdoc Hesam Askari (now at the University of Rochester) devised the simplest continuum-mechanics equations that could describe granular flow around an object that intrudes into a medium. They then fed those equations into numerical simulations.[3] Their investigation uncovered the scaling laws that determine RFT's validity. Ultimately, their insight could save RFT practitioners time and extend the theory's range to other media.

Swimming in sand

Granular RFT got its start in 2008 when Daniel Goldman of Georgia Tech in Atlanta noticed that sandfish wriggle through sand like nematode worms swim through fluids. Goldman's PhD student Yang Ding (now at the Beijing Computational Science Research Center) suggested trying out RFT to model the lizard's motion. They knew that RFT when applied to viscous fluids had limitations, but, says Goldman, "We were naive and young and just said, well, why not."

In essence, RFT states that the forces on any small element of a moving body are independent of the motions and po-

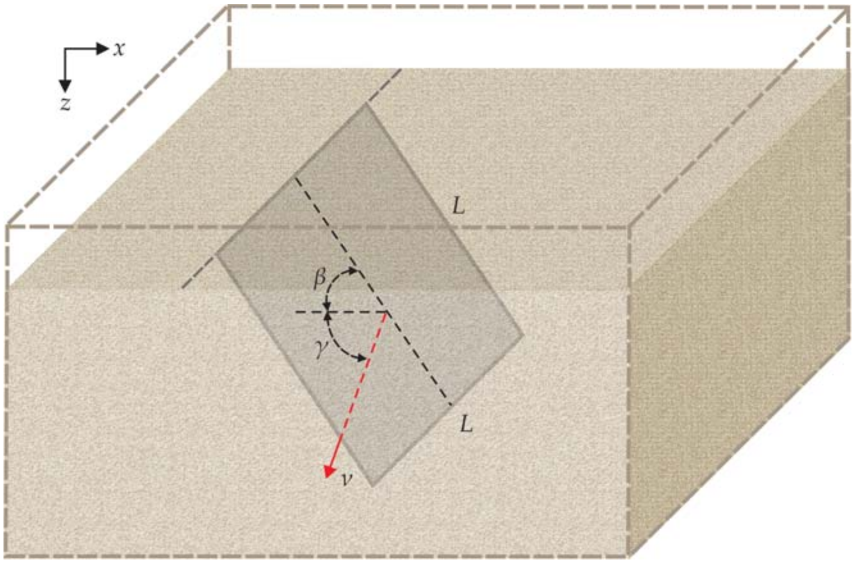


FIGURE 2. A SQUARE PLATE submerged in a medium up to its top edge is made to move in the *xz* plane at speed *v*. The top edge of the plate is perpendicular to the *xz* plane so a single angle *β* relative to the horizontal is sufficient to specify the plate's orientation. The direction of motion is given by the angle *γ*. The validity of resistive force theory can be tested by checking to see if its results correctly scale with the plate's size. (Adapted from ref. 3.)

sitions of other body elements. (See the Quick Study by Yang Ding, Chen Li, and Daniel Goldman, PHYSICS TODAY, November 2013, page 68.) Thus the net force on the body is simply a superposition of forces on infinitesimal elements. To apply RFT to a wriggling sandfish swimming at constant speed, one sets the net force to zero and calculates the speed consistent with that requirement.

In viscous fluids, RFT is an approximation of the well-established Stokes equations that determine stress and flow fields in fluids. So the forces on infinitesimal segments of a swimming nematode can, in principle, be calculated from scratch.

No such constitutive equations existed for sand and other granular media. So Goldman, Ding, and their colleagues fabricated stainless steel cylindrical rods with roughly the same frictional properties as sandfish skin to serve as proxies for sandfish body segments. They measured the resistive forces on the cylinders as they moved them through sand at different orientations and in different directions. They could then use the measurements as inputs for RFT calculations.

In 2010 Kamrin saw Goldman give a talk on the sandfish study and was intrigued. "I couldn't believe how well it worked," he says. By 2013 Goldman's group had shown that RFT also works for insects running on sand, and Kamrin wanted to know why. "It seemed clear to me that this was a problem worth solving at the fundamental level."

Back at MIT, Kamrin enlisted Askari to help develop a theory that can explain RFT's success. A natural approach to take—and one that's commonly applied to granular material—might have been the discrete element method (DEM), which can be used to simulate the motions of millions of individual grains. But the method is computationally intensive and, Kamrin and Askari explain, the variables are so numerous that it can be difficult to derive a general theory from the results. They instead opted to see if RFT emerges from a simple continuum theory. The only inputs to their continuum model were the medium's density and internal friction coefficient.

Tests and analysis

Kamrin and Askari did check their model's predictions against published DEM results.[4] For the case of a thin, flat plate intruding into granular material, their model reassuringly reproduced the flow patterns in the material that DEM simulations did.

They then used their model to put RFT through a battery of tests. For the flat-plate scenario, they used their model to create resistive force plots—maps of predicted force profiles as functions of the plate's orientation and direction of motion. They compared their plots against those produced experimentally and found good agreement.

The two theorists also looked at intruders of different shapes. They tested a cylinder, a rectangular prism, and a V-shaped object. The match between RFT predictions and their continuum model was so good, Askari says, "it was getting a little scary."

All that raises the question, Why does RFT work so well? Because of the simplicity of a square plate's geometry, Kamrin and Askari could gain insight from dimensional analysis. From the equations built into their model, the analysis implied that the granular resistive force on a square plate of width L, illustrated in figure 2, must scale as L^3.

Kamrin and Askari further reasoned that if RFT works, they could shrink the plate width from L to a much smaller value λ, and then calculate the resistive force on the L-sized plate by summing the forces on a collection of λ-sized plates. And indeed, for granular media, RFT produces the correct scaling. In fact, for a square plate, it predicts the force exactly.

However, if one does the same analysis for a plate moving through a viscous fluid, RFT doesn't scale correctly. The

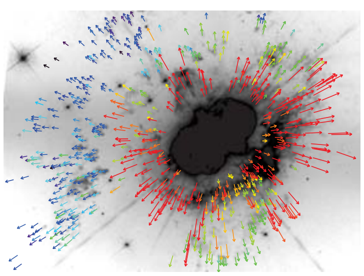
PHYSICS UPDATE

These items, with supplementary material, first appeared at www.physicstoday.org.

SUPERNOVA IMPOSTOR PEGGED AS REPEAT OFFENDER

For a short spell in the mid 1800s, η Carinae, a hypergiant star in the Southern Hemisphere's Carina constellation, had the look of what astronomers now call a supernova: It ejected some 15 solar masses of gas and dust, released 10^{43} joules of energy, and briefly became the second-brightest star in the sky. But unlike a star undergoing a supernova, η Carinae survived the flare-up. The event, known as the Great Eruption, has long puzzled astronomers, and now a study by Megan Kiminki, Nathan Smith (both at the University of Arizona), and Megan Reiter (University of Michigan) suggests there's more to η Carinae's story.

By analyzing 20 years of *Hubble* images, Kiminki and her coworkers were able to estimate the velocities of hundreds of objects in the expanding cloud of ejecta that surrounds the star. Extrapolating the trajectories back in time, they could infer the moment, give or take a few decades, when each object was hurled from the star. The results indicate that the ejecta couldn't have originated from a single eruption; there had to be two others—one in the 1500s and another in the 1200s. In the image, the colors of the velocity arrows indicate the time of an object's ejection (red, 1800s; yellow–green, 1500s; blue–violet, 1200s). The star itself is obscured by an oblong cloud known as the Homunculus Nebula. The apparent 300-year cycle of outbursts rules out at least one popular explanation for the Great Eruption—that it was a one-time event triggered when η Carinae swallowed a smaller companion star—and places new constraints on others. (M. M. Kiminki, M. Reiter, N. Smith, *Mon. Not. R. Astron. Soc.* **463**, 845, 2016.)



—AGS

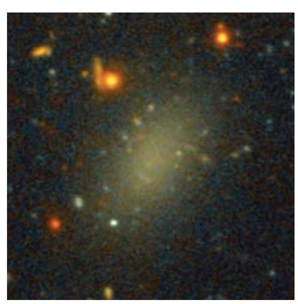
GRAPHENE MEMBRANES' ANOMALOUS DYNAMICS

The burgeoning exploration and development of the nanoworld is forcing scientists to reexamine and reframe many established aspects of our familiar macroscopic world. Among the tenets experiencing renewed interest is the nature of the thermal fluctuations that underlie the random walks of Brownian motion (see, for example, the Quick Study by Mark Raizen and Tongcang Li, PHYSICS TODAY, January 2015, page 56). The University of Arkansas's Paul Thibado and colleagues now report ultraprecise dynamical measurements on a freestanding, atomically thin sheet of graphene that show clear deviations from classical Brownian behavior. With its regular hexagonal lattice, monolayer graphene offers an exceptionally clean two-dimensional system for studying membrane behavior. Using a custom scanning tunneling microscope (STM), the researchers tracked the out-of-plane dynamics of an atom-sized region of the membrane with subnanometer, millisecond resolution for more than two and a half hours. Over that time, the region's height exhibited Brownian excursions that spanned 10 nm, but they were punctuated with rare, large jumps—so-called Lévy walks (see the article by Joseph Klafter, Michael Shlesinger, and Gert Zumofen, PHYSICS TODAY, February 1996, page 33). Molecular dynamics simulations revealed that the jumps arose from spontaneous mechanical buckling, which changed the region's local curvature and height. The graphene studies may yield insights into the contributions thermal fluctuations make to the proper functioning of biomembranes. Moreover, the STM could be used to tune the graphene's stochastic fluctuations, offering the tantalizing prospect of artificial nanoscale motors and engines. (M. L. Ackerman et al., *Phys. Rev. Lett.* **117**, 126801, 2016.)

—RJF

A LARGE GALAXY MADE ALMOST ENTIRELY OF DARK MATTER

Scattered among the galaxies of the Coma cluster are dozens of objects that are as large as the Milky Way but shine only 1/100 as bright. The stability of those ultradiffuse galaxies (UDGs) in the tidally disruptive Coma environment suggests that they may be held together by an unusually large component of dark matter. Now a research team led by Pieter van Dokkum of Yale University has measured the mass of one UDG, Dragonfly 44, and found a value similar to that of our Milky Way. Combining mass and luminosity determinations, the group concluded that Dragonfly 44 is 99.99% dark matter.



The group's key measurement was of the UDG's spectrum near the hydrogen-alpha absorption line at 656 nm. Because different stars in the galaxy move with different speeds, the nominal absorption line is broadened to a peak whose width can be related to the mass of the central, star-containing portion of the galaxy. In essence, greater mass yields stronger centripetal forces, a wider range of speeds, and a broader peak. The spectral measurements plus modeling yielded an estimate for mass in the dark-matter halo of Dragonfly 44 and thus for the galaxy's total mass.

Stokes equations establish that the force must be proportional to L, but the sum delivers a net force that scales as L^2.

Kamrin and Askari call their analysis the garden hoe test, after the square plate's resemblance to the familiar tool. They envision applying it to predict how well RFT will perform for other types of media. For instance, the test shows that for certain types of gels, pastes, and muds—media that produce drag forces proportional to L^2—RFT correctly predicts the forces.

In addition, the pair explains that the agreement between granular RFT and their continuum model means that the model can be used to generate inputs for RFT calculations. That could spare researchers the labor-intensive force measurements currently required. And RFT could be used for off-Earth or other environments that can't easily be reproduced in the lab.

Sung Chang

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HOW TYPHOONS CHANGE THE UNDERWATER SOUND FIELD

A typhoon or hurricane pushes surface water before it. When the wind-driven water hits the coast, it can no longer move forward. Forced downward and backward, the surface water displaces deeper, colder water and sends it far out to sea. Because the speed of sound in seawater rises with temperature, a typhoon's passage alters the coastal sound field. Now Guang-Bing Yang and his colleagues from the First Institute of Oceanography in Qingdao, China, have identified a second, related effect to do with seafloor sediment. On 1 August 2012, one day before Typhoon Damrey (shown here) made landfall in eastern China, the crew of the fishing boat *Lulaoyu* took the temperature profile of the water at a location 10 km off the coast of Qingdao. Four days later, after the typhoon had left, the *Lulaoyu* returned to take a second profile. When Yang and his collaborators analyzed the profiles, they found that water at the maximum depth of 32 m had risen in temperature by 5 °C. According to calculations by Yang and his collaborators, heat from the warmer water diffused through the sediment and raised the temperature there and, with it, the sound speed—by up to 15 m/s. To quantify that effect, the researchers modeled the case of a sound source positioned 1 m above the seafloor.

NASA/LANCE MODIS RAPID RESPONSE TEAM

For a distance up to 16 km from the coast and for four days after a typhoon, the warmer sediment changed the acoustic power by at least 10 dB. (G.-B. Yang et al., *J. Acoust. Soc. Am.* **140**, EL242, 2016.) —CD

In addition to having a mass near that of the Milky Way, Dragonfly 44 encompasses nearly 100 globular clusters, an anomalously large number for such a dim galaxy. The large mass and cluster population of Dragonfly 44, says van Dokkum's team, seem to rule out scenarios, plausible for lighter UDGs, in which the galaxy started as a conventional one that subsequently puffed up. Instead, the researchers argue, Dragonfly 44 is more likely a "failed" galaxy in which supernova explosions or some other process somehow squelched star formation at an early age. (P. van Dokkum et al., *Astrophys. J. Lett.* **828**, L6, 2016.) —SKB

WATER FLOWS FREELY THROUGH CARBON NANOTUBES

Despite the frenzy of research into carbon nanotubes (CNTs) over the past few decades (see, for example, the article by Thomas Ebbesen, Physics Today, June 1996, page 26), there isn't much experimental evidence for one of the tiny structures' most talked-about superpowers: the ability to funnel water with nearly zero friction. The problem has been achieving the sensitivity to measure water transport rates as feeble as a femtoliter a second. Now Lydéric Bocquet and his colleagues at École Normale Supérieure in Paris have confirmed the slipperiness of CNTs by directly measuring water flow through individual nanotubes whose bores ranged from 15 nm to 50 nm. The researchers stuck a multiwalled CNT inside a small pipette and essentially turned the nanotube into the needle of a syringe. Pressure applied inside the pipette caused water to flow through the CNT and into a tank of water. Rather than tracking the water as it flowed through the tube, Bocquet and his team analyzed the motion of suspended polystyrene nanobeads in the tank to deduce the strength of the jet emerging from the CNT (see the image, which shows the response at various pressures). The results verify that CNTs allow water to flow extremely efficiently. Bocquet's team also confirmed its 2010 prediction that the flow rate would increase as the tube's radius decreased, although the dependence turned out to be roughly quadratic rather than quartic. The biggest surprise came when the researchers replaced the CNTs with nanotubes of boron nitride. Although the BN tubes are nearly structurally identical to their carbon counterparts (see the article by Marvin Cohen and Alex Zettl, Physics Today, November 2010, page 34), they proved far more resistant to water flow. The finding seems to suggest that electronic properties—CNTs are conductors; boron nitride nanotubes are insulators—play a role in hydrodynamics at very small scales. Bocquet and his team plan to investigate that possibility as they explore the nanotubes' potential for applications such as water distillation and filtration. (E. Secchi et al., *Nature* **537**, 210, 2016.) —AG PT