Effect of volume fraction on granular avalanche dynamics

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We study the evolution and failure of a granular slope as a function of prepared volume fraction, \( \phi_0 \). We rotated an initially horizontal layer of granular material (0.3-mm-diam glass spheres) to a 45\(^\circ\) angle while we monitor the motion of grains from the side and top with high-speed video cameras. The dynamics of grain motion during the tilt process depended sensitively on \( \phi_0 \in [0.58–0.63] \) and differed above or below the granular critical state, \( \phi_c \), defined as the onset of dilation as a function of increasing volume fraction. For \( \phi_0 - \phi_c < 0 \), slopes experienced short, rapid, precursor compaction events prior to the onset of a sustained avalanche. Precursor compaction events began at an initial angle \( \theta_0 = 7.7 \pm 1.4^\circ \) and occurred intermittently prior to the onset of an avalanche. Avalanches occurred at the maximal slope angle \( \theta_m = 28.5 \pm 1.0^\circ \). Granular material at \( \phi_0 - \phi_c > 0 \) did not experience precursor compaction prior to avalanche flow, and instead experienced a single dilational motion at \( \theta_0 = 32.1 \pm 1.5^\circ \) prior to the onset of an avalanche at \( \theta_m = 35.9 \pm 0.7^\circ \). Both \( \theta_0 \) and \( \theta_m \) increased with \( \phi_0 \) and approached the same value in the limit of random close packing. The angle at which avalanching grains came to rest, \( \theta_R = 22 \pm 2^\circ \), was independent of \( \phi_0 \). From side-view high-speed video, we measured the velocity field of intermittent and avalanching flow. We found that flow direction, depth, and duration were affected by \( \phi_0 \), with \( \phi_0 - \phi_c < 0 \) precursor flow extending deeper into the granular bed and occurring more rapidly than precursor flow at \( \phi_0 - \phi_c > 0 \). Our study elucidates how initial conditions—including volume fraction—are important determinants of granular slope stability and the onset of avalanches.

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Granular materials are collections of discrete particles that interact through repulsive contact forces [1–3]. These materials are of interest to physicists and engineers largely because of their ability to transition between fluidlike and solidlike states [2]. A granular avalanche is an important example of the granular solid-to-fluid transition. Avalanches of granular media are ubiquitous, occurring in industrial [4], laboratory [5], and natural settings (landslides).

The packing of a bed of granular material, measured by the prepared volume fraction, \( \phi_0 \), may vary from loosely packed (low \( \phi_0 \)) to closely packed (high \( \phi_0 \)) initial conditions. \( \phi_0 \) is defined as the ratio of solid volume to the occupied volume, and for relatively monodisperse particles \( 0.57 < \phi_0 < 0.64 \) [3]. A granular material’s resistance to flow is significantly affected by \( \phi_0 \), as observed in boundary shear experiments [6–8], triaxial tests [9,10], and localized forcing by submerged objects [11–13]. In general, loosely packed granular material compacts when a shear stress or strain is applied while closely packed granular material dilates under shear [6]. The volume fraction at which the failure response of granular material transitions from compaction to dilation is called the granular critical state, \( \phi_c \), and it separates the two modes of granular failure under boundary-imposed forcing [6].

Little is known about how \( \phi_0 \) influences the failure and avalanche onset of a dry granular slope. This is largely because in most granular avalanche experiments, the initial transient behavior is removed. For example, avalanching studies in continuously rotating drums [5,14–16], or particle deposition onto a slope [17], are typically performed in a steady state, such that transient behaviors are completed before observations are made. Thus in such experiments, the influence of \( \phi_0 \) is not the focus of study.

An alternative method of investigating avalanche dynamics is through progressive loading. In such experiments, an undisturbed granular bed slowly rotates from a horizontal

FIG. 1. (Color online) Experiment setup and volume fraction evolution during tilting of the granular bed. (a) Air flow through a porous floor in the fluidized bed and mechanical vibration prepare granular material to the desired initial \( \phi \). The bed rotates about midpoint and is imaged from the top and side views. The coordinate system is in the frame of reference of the rotating bed. (b) The average instantaneous volume fraction of the granular bed \( \phi(t) \) vs tilt angle, \( \theta \), for 14 experiments at different \( \phi_0 \). As the layer is inclined, compaction or dilation precursor events precede an avalanche. The preavalanche and failure dynamics are sensitive to initial \( \phi_0 \).

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An alternative method of investigating avalanche dynamics is through progressive loading. In such experiments, an undisturbed granular bed slowly rotates from a horizontal
orientation until an avalanche occurs [18]. In this way, granular materials can be prepared to a predetermined \( \phi \) prior to rotation. However, the majority of progressive loading experiments do not control \( \phi \). Progressive loading experiments typically performed at a single prepared \( \phi \approx 0.6 \) (the “as-poured” state) have found that small rearrangements of the granular layer (precursors) precede the eventual surface avalanche onset [19–22]. Previously, volume fraction has been found to be an important determinant of the angle of maximum stability at which a granular avalanche begins [23]. Furthermore, a recent study of the precursor events in progressive loading of dry granular material reported that the intensity of these precursor events differed between two granular preparations, a densely and loosely packed initial state [22], suggesting an importance of \( \phi \) on slope dynamics.

The effect of \( \phi \) on avalanche onset has been systematically studied in granular suspension experiments in which the granular material is immersed in a fluid. Results from these experiments [22,24,25] give an indication of what we may expect in dry granular materials: increasing \( \phi \) increases the angle of maximum stability. However, granular suspensions, with the presence of a viscous interstitial fluid, differ from dry granular materials. The feedback between fluid pore pressure and granular flow can be significant in suspension experiments, and can alter the mode of failure from that of dry granular materials. Additionally, frictional interactions among grains with an intermediate fluid may also differ from the dry case. A final difference between the previous suspension experiments and our current study is that the volume fractions realized in suspensions (\( \approx 0.56–0.59 \)) are typically lower than those of dry granular material (\( \approx 0.57–0.64 \)).

In this paper, we vary the prepared volume fraction of a dry granular material in experiment, and study the pre- and post-avalanche dynamics. We find that both pre- and post-avalanche behavior is sensitive to \( \phi \), and both exhibit a change in stability as \( \phi \) is increased above the critical state volume fraction, \( \phi_c \).

I. MATERIALS AND METHODS

An air fluidized bed of length \( l = 43 \) cm and width \( w = 28 \) cm was filled to a depth of \( d \approx 9 \) cm (bed height varied with packing condition) with a granular material of \( D = 256 \pm 44 \mu m \) diameter spherical glass beads [Fig. 1(a)]. Air flow through a porous rigid floor in the bed fluidized the granular material, and a combination of flow and mechanical vibration controlled the volume fraction as a function of vibration duration [12]. The volume fraction is defined as \( \phi(t) = \frac{M}{\rho g h(t)} \), where \( M \) is total grain mass, \( \rho = 2.5 \, \text{g/cm}^3 \) is grain density, and \( h(t) \) is the time-varying height of the granular material. The prepared volume fraction is defined as \( \phi_0 = \phi(t = 0) \). We varied the duration of mechanical vibration during granular state preparation to vary the initial \( \phi_0 \) between 0.58 < \( \phi_0 < 0.63 \). We defined the total change in volume fraction prior to the onset of an avalanche, \( \Delta \phi = \phi_f - \phi_0 \).

The experiments consisted of a constant rotation of the bed at angular speed 2.1 deg/s from an initial angle \( \theta = 0^\circ \) to a final angle of \( \theta = 45^\circ \). We performed similar experiments at lower rotation rates (down to 0.8 deg/s), but did not observe any significant change in avalanche dynamics. Two cameras recorded the granular motion during bed rotation. One camera mounted above the granular surface recorded grain motion at 30 Hz. A second camera was mounted on the side of the bed and imaged the granular material adjacent to the transparent wall at 200 Hz. The observation region of the side-view camera was 6 \times 8 \, \text{cm}^2 (240 \times 320 \, \text{D}^2) in height and width and the camera was centered in the middle of the length of the bed [see Fig. 1(a)]. We back-lit the granular bed such that the granular surface was detected as a high-contrast edge from black (grains) to white (background). To determine when flow occurred, we evaluated difference images between consecutive image frames. The pixel intensity in difference images did not map linearly to the amount of deformation that occurred between frames because the granular surface has similar texture features. However, difference images were a repeatable method of measuring when grain configurations changed, which we confirmed with follow-up particle image velocimetry experiments.

In addition to experiments, we recorded video from the side at 1000 Hz; these images were used to find the velocity profiles of the particle flow at the sidewall through particle image velocimetry (PIV). This PIV method has been previously used in studying granular flows during plate drag [11,12]. Image resolution was 251 \( \mu m \) per pixel in both dimensions. We used a custom PIV algorithm that measured image correlations using the method of [26] with 1/10th subpixel resolution. We ignored PIV measurements at the surface where the stationary background hindered the correlation. We measured the depth-averaged PIV flow values for the horizontal and vertical granular flow directions over the depth range of 0–180 D.

The surface profile of the granular bed was tracked as a function of rotation angle over the duration of the experiment. The average height of the granular surface, \( h(t) \), was used to compute the instantaneous mean volume fraction \( \phi(t) \) by the equation given above. We defined \( \theta_0 \) as the angle of the bed at which the first motion of the granular slope is observed. We define an avalanche as a continuous flow of granular material that resulted in a change in granular slope, \( \theta_m \) was defined as the bed angle at which there occurred a continuous flow (lasting in duration greater than 1 s) in the material, which we defined as an avalanche. Lastly, \( \theta_R \) is the repose angle of the granular material—the angle of the granular surface after the avalanche halts. We monitored the bed angle by recording the voltage drop across a potentiometer within the actuator.

II. \( \phi \)-DEPENDENT SLOPE FAILURE

Granular slope response during tilting differed as a function of \( \phi_0 \). At low \( \phi_0 \), we observed that the slope underwent several compaction events—precursors—prior to the onset of avalanching flow [Fig. 1(b)]. Precursor compaction events were observed as a rapid increase in \( \phi(t) \) and occurred intermittently throughout the tilting process and prior to the formation of an avalanche [Fig. 1(b)]. As \( \phi_0 \) was increased, we observed that the tendency for granular precursors to occur prior to an avalanche diminished. At large \( \phi_0 \), we found that the granular slope underwent dilatation [a decrease in \( \phi(t) \)] immediately prior to avalanching flow at \( \phi_f \).
The total change in volume fraction from initial grain motion to the first avalanche is $\Delta \phi = \phi_f - \phi_0$. $\Delta \phi$ decreased from a positive value to a negative value as $\phi_0$ increased. The critical state volume fraction where no dilation or compaction occurred was $\phi_c = 0.595 \pm 0.003$. Our measurement of $\phi_c$ is in accord with previous measurements made in plate drag experiments [11]. However, we note that the value of $\phi_c$ depends upon the material properties and the shape of the grains, in addition to the applied stresses on the grains [8], and thus varies across materials and experiments.

For $\phi_0 < \phi_c$, granular material compacted prior to the onset of an avalanche, and for $\phi_0 > \phi_c$, granular material dilated prior to the onset of an avalanche [Fig. 2(a)]. For $\phi_0 > \phi_c$, increasing $\phi_0$ resulted in $\Delta \phi$ decreasing to a smaller yet still negative value. This indicated that the magnitude of dilation preceding an avalanche decreased with increasing $\phi_0$. Such a decrease in $\Delta \phi$ likely occurs because higher $\phi_0$ granular material experience larger internal stresses, which may inhibit volumetric expansion.

Both $\theta_0$ and $\theta_m$ increased with increasing $\phi_0 - \phi_c$ [Fig. 2(b)]. Avalanche precursors occurred at low angles for low $\phi_0$, with $\theta_0 = 7.7 \pm 1.4^\circ$ in the case of $\phi_0 - \phi_c = -0.006 \pm 0.001$. As $\phi_0 - \phi_c$ increased, so did $\theta_0$, and for closely packed granular material, $\phi_0 - \phi_c = 0.032 \pm 0.001$, we observed $\theta_0 = 32.1 \pm 1.5^\circ$.

The bed angle at avalanche onset, $\theta_m$, was also a function of $\phi_0 - \phi_c$ [Fig. 2(b)]. Increasing $\phi_0 - \phi_c$ increased $\theta_m$ from $\theta_m = 28.5 \pm 1.0^\circ$ to $35.9 \pm 0.7^\circ$ over the range of volume fractions observed. Although $\theta_m$ was sensitive to $\phi_0$, the difference in magnitude of $\theta_m$ over the observed $\phi_0$ was not as large as the variation observed in $\theta_0$. We expect that the decreased sensitivity of $\theta_m$ on $\phi_0$ is due to the series of precursor events that occur during the tilting process and prior to avalanche flow for $\phi_0 < \phi_c$, which likely strengthen the material. Our results for $\theta_m$ are in accord with a previous study of granular avalanches of Hostun sand, in which $\theta_m$ varied from $\approx 26^\circ$ to $37^\circ$ over a range of five different initial packings from loose to close [23].

The final slope angle at which the granular material came to rest after the full $45^\circ$ rotation is defined as the angle of repose, $\theta_R$. Over the range of $\phi_0$ observed, $\theta_R$ was independent of $\phi_0 - \phi_c$ [Fig. 2(c)], with a value of $\theta_R = 22 \pm 2^\circ$. The lack of dependence of $\theta_R$ on $\phi_0 - \phi_c$ indicates that due to the grain motion prior to and during an avalanche the granular material evolves to a critical state, independent of its previous state [6]. The independence of $\theta_R$ on prepared volume fraction is evidence that $\theta_R$ is a property of the granular material, dependent only on the grain mechanical properties (coefficient of friction, restitution, shape, etc.) [27].

The angle of repose we observe for dry, spherical glass beads is consistent with previous observations [28]. Additionally, the values of $\theta_m$ we observe are consistent with the ranges reported in previous experiments on similar granular material [28]. However, our observations of granular motion at tilt angles lower than $\theta_0 < 10^\circ$, which occur at low $\phi_0 - \phi_c$, have not been observed in previous granular avalanche experiments. It is likely that in previous experiments in which $\phi_0 - \phi_c$ was not varied, the volume fraction was near $\phi_c$ and was in the “as-poured” granular state. The dynamics of failure and flow away from $\phi_c$ are less understood, and our results shed light on this phenomenon.

III. GRANULAR FLOW AT FAILURE

To characterize the granular flow during the precursor events at varying $\theta_0$, we computed the flow profile at a sidewall using PIV (Fig. 3). Since precursor events consisted of a flow initiation and flow arrest, we measured the displacement field of the granular flow $[d_x(y), d_y(y)]$ along a vertically oriented line centered in the imaging region and in the reference frame of the tilted bed. The displacement field of the granular flow during the first precursor depended on $\phi_0 - \phi_c$. In loose granular media ($\phi_0 - \phi_c < 0$; see Fig. 3(a)), flow resulted in material displacement parallel to and down the length of the bed [positive $d_x(y)$], and a vertical displacement toward the floor of the enclosure [positive $d_y(y)$]. Positive $d_y(y)$ indicates a compaction of the granular material as expected for $\phi_0 - \phi_c < 0$. When the granular material was prepared near the critical state, $\phi_0 - \phi_c \approx 0$ [Fig. 3(b)], $d_x(y)$ decreased and $d_y(y)$ approached zero as compared to the loose pack state. This indicated that no compaction or dilation occurred during failure onset—the definition of the granular critical state. Lastly, in the dilating regime, $\phi_0 - \phi_c > 0$ [Fig. 3(c)], $d_x(y)$ decreased as $\phi_0$ increased and $d_y(y)$ was negative and
For our geometry, we assume translational invariance along may estimate the dilatancy angle as \( \tan(\psi) \), makes with the horizontal. In the case of simple shear, we the angle the displacement vector of the surface deformation is measured by the tangent of the dilatancy angle, \( \phi_c \) of the function of reference of the granular bed, and the downslope direction is to now evaluate quantitative differences in granular response in [Figs. 4(a) and 4(b)], as expected from the definition of the critical state. The difference in magnitude of \( \phi_c \) for \( \phi_0 - \phi_c > 0 \). At higher volume fractions—above \( \phi_c \)—initial grain motion resulted in comparatively small displacements during precursor failure [Figs. 4(a) and 4(b)].

The dilation that occurs per unit shear in a granular material is measured by the tangent of the dilatancy angle, \( \psi \) [Fig. 4(c)]. For our geometry, we assume translational invariance along the horizontal direction (x in the frame of the rotating bed), similar to a simple-shear flow. The dilatancy angle therefore is the angle the displacement vector of the surface deformation makes with the horizontal. In the case of simple shear, we may estimate the dilatancy angle as \( \tan(\psi) = \frac{d_y}{d_x} \) averaged over the entire depth of the flow. We plot \( \tan(\psi) \) as a function of \( \phi_0 - \phi_c \) [Fig. 4(c)] and we find that the dilatancy angle during precursor failure obeys the equation \( \tan(\psi) = K(\phi_0 - \phi_c) \), where \( K = 22.4 \pm 2.4 \). This relationship was originally introduced by Roux et al. [29] and is effectively a linearization of the function \( \psi(\phi_0 - \phi_c) \) about the critical state \( \phi_0 - \phi_c = 0 \).

To gain insight into the values of \( \tan(\psi) \) observed in experiment, we construct a simple scenario of dilating and compacting flow. We imagine a two-dimensional arrangement of diameter \( D \) spheres in a square lattice (loose-packing) or a hexagonal lattice (close-packing). In both cases, we impose a lateral displacement, \( d_x \), and we compute the resultant \( d_y \) of the grain layer [see the drawings in Fig. 4(c)]. In both cases, for a lateral displacement of \( d_x = \frac{D}{2} \), the grains must displace maximally vertically \( d_y = 0.134 \) in the \( \pm y \) direction, resulting in \( \tan(\psi) = \pm 0.268 \) [where by definition (\( \pm \)) is dilation]. The peak dilation and compaction observed in experiment are close to the values of \( \tan(\psi) \) predicted from the simple scenario presented above, which gives some insight into the magnitude of compaction (dilation) that the loose (close) pack undergoes in experiment [see the gray lines in Fig. 4(c)]. Larger and smaller values of \( \tan(\psi) \) are likely due to the disordered nature of the granular configurations created in experiment.

Finally, we compare our results on dry granular materials to those on granular suspensions performed by Palha et al. [24]. They observed a similar linear relationship between \( \phi_0 \) and \( \tan(\psi) \) [Fig. 4(c)], with fit parameters of \( K = 3.4 \) and \( \phi_c = 0.582 \). The lower value of \( \phi_c \) observed in [24] is likely due to the hydrostatic fluid pressure in their experiments, which reduces internal granular stresses and thus reduces \( \phi_c \) [8]. A lower value of \( K \) indicates that for comparable horizontal grain
motion, less vertical motion is experienced in the granular suspension compared to the dry granular material. The smaller vertical motion in the fluid-immersed experiments [24] is likely due to the resistive influence of the pore pressure from the surrounding viscous fluid. For instance, a suction pressure is generated when a fluid-immersed granular material dilates, generating higher grain-grain stresses that in turn may resist the dilation motion, and thus lower $d_c$.

**IV. $\phi_0$ Dependence on Flow Depth**

During the granular flow at $\theta_0$, we observed that the flow profile extended deeper into the granular layer than during avalanche flow at $\theta_m$. We observed this by computing image differences of successive video frames from high-speed video (1000 Hz) of the progressive loading experiment [Fig. 5(a)]. We visualized the dynamics of the avalanche process by examining the space-time evolution of the image difference magnitude, evaluated along a thin vertical strip centered in the observation region [white boxes in Fig. 5(a)]. We constructed space-time images by plotting the intensity of difference images (time interval of 1 ms) at depth, $y$, and time, $t$, within this thin strip. Image difference profiles for high and low $\phi_0$ showed intermittent precursor events at $\phi_0 - \phi_c < 0$ prior to an avalanche [Fig. 5(b)]. As $\phi_0$ increased, the angle at which the precursor flow occurred also increased.

The space-time evolution of individual precursor events varied in shape and size as a function of $\phi_0 - \phi_c$ [Figs. 6(a) and 6(b)]. We observed that precursor events either initiated from the surface and propagated downward [Fig. 6(a)] or occurred simultaneously throughout the layer [Fig. 6(b)]. However, we did not observe a systematic dependence of this behavior on $\phi_0 - \phi_c$. In the example shown in Fig. 6(a), in which the flow-front propagated downward, the propagation speed of the compaction front was $v = 1.08$ m/s. For comparison, the speed of sound propagation in granular material is $\approx 280$ m/s [30] and the speed of sound in glass is $\approx 4000$ m/s.

Although the spatiotemporal evolution of the precursor flow did not vary with $\phi_0 - \phi_c$, the magnitude of spatiotemporal flow events varied with $\phi_0 - \phi_c$ in both maximum depth ($d_p$, for avalanche depth and $d_p$ for precursor avalanche depth), and precursor flow duration, $\Delta T$ [Figs. 6(c) and 6(d)]. $\Delta T$, was a nonmonotonic function of $\phi_0 - \phi_c$ and increased to a maximum near $\phi_c$ [Fig. 6(c)]. Precursor depth was sensitive to $\phi_0 - \phi_c$ and was approximately constant, $d_p = 320 \pm 2.4$ D, for $\phi_0 - \phi_c < 0$ and linearly decreasing, $d_p = (-968 \text{ D})\phi_0 + 33$ D, for $\phi_0 - \phi_c > 0$ [Fig. 6(d)]. The functional difference in $d_p(\phi_0)$ as $\phi_0$ exceeds $\phi_c$ is another signature of a bifurcation in the granular rheology that occurs at the dilation transition [11].

The avalanche depth, $d_a = 45 \pm 17$ D, was significantly shallower than the precursor events [Fig. 6(d)] and was independent of $\phi_0 - \phi_c$. The independence of $d_a$ as a function of $\phi_0 - \phi_c$ suggests that fully formed avalanche flow is not dependent upon the prior initial state. Insensitivity of avalanche depth likely occurs because grain rearrangements prior to
FIG. 6. (Color online) Spatiotemporal evolution of granular flow during precursors and avalanches. (a) and (b) Space-time images of precursor events at $\theta_0$ in a loose- (a) and close- (b) packed granular media. In example (a), flow begins at the surface and propagates vertically into the granular layer (black arrow) at a speed of approximately $v = 1.08$ m/s, to a maximum depth $d_p$. In example (b), flow begins simultaneously throughout the layer. The intensity of the image is arbitrary and chosen to highlight flow shape. (c) Time duration of precursor events as a function of $\phi_0 - \phi_c$. (d) Depth dependence of precursor and avalanche flow as a function of initial $\phi_0 - \phi_c$. White circles are precursor flow and black circles are avalanching flow. The black and green lines show the fit functions described in the text.

avalanche formation act to compact or dilate the granular material toward the critical state, independent of $\phi_0$.

V. CONCLUSION

Understanding the conditions that lead to failure of a granular slope have important applications in diverse fields such as bridge and dam design, the development of rugged robots [31] and extraterrestrial rovers [32], and the biology of sand-dwelling organisms [33]. In our experimental investigation of the effect of volume fraction on the failure of a granular slope, we found that slope failure differed significantly as a function of initial $\phi_0$.

Response of the granular media to our progressive loading experiment was divided into roughly two regimes, above or below $\phi_c$. Below $\phi_c$, the granular material underwent several compaction events prior to the onset of sustained surface flow defined as an avalanche. The angle at which compaction precursor events began in the case of the lowest $\phi_0$ occurred at $\theta_0 = 7.7 \pm 1.4^\circ$, a value of slope failure substantially lower than what has been observed in previous avalanche experiments with “as-poured” granular material in which volume fraction is not typically varied. As $\phi_0$ was increased, we observed an increase in $\theta_0$, which approached the maximum angle of stability, $\theta_m$, as $\phi_0$ increased. The value of $\theta_m$ we observed for the critical state, $\phi_c$, was consistent with values observed for similar glass particles prepared in an “as-poured” state or subject to multiple avalanches in rotating drum experiments [28]. We emphasize that, as in previous experiments [8,11,13,23,34], $\phi_0$ plays an important role in granular flow in response to stress.

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