Dynamics of scattering in undulatory active collisions

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Natural and artificial self-propelled systems must manage environmental interactions during movement. In complex environments, these interactions include active collisions, in which propulsive forces create persistent contacts with heterogeneities. Due to the driven and dissipative nature of these systems, such collisions are fundamentally different from those typically studied in classical physics. Here we experimentally and numerically study the effects of active collisions on a laterally undulating sensory-deprived robot, whose dynamics are relevant to self-propelled systems across length scales and environments. Interactions with a single rigid post scatter the robot, and this deflection is dominated by head-post contact. These results motivate a model which reduces the snake to a circular particle with two key features: The collision dynamics are set by internal driving subject to the geometric constraints of the post, and the particle has an effective length equal to the wavelength of the snake. Interactions with a single row of evenly spaced posts (with interpost spacing \( d \)) produce distributions reminiscent of far-field diffraction patterns: As \( d \) decreases, distinct secondary peaks emerge as large deflections become more likely. Surprisingly, we find that the presence of multiple posts does not change the nature of individual collisions; instead, multimodal scattering patterns arise from multiple posts altering the likelihood of individual collisions to occur. As \( d \) decreases, collisions near the leading edges of the posts become more probable, and we find that these interactions are associated with larger deflections.

Our results, which highlight the surprising dynamics that can occur during active collisions of self-propelled systems, can inform control principles for locomotors in complex terrain and facilitate design of task-capable active matter.

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I. INTRODUCTION

Biological and artificial systems must manage mechanical interactions with the environment to generate and sustain movement. These interactions come in myriad forms, from repeated impacts with rigid ground [1] to managing and manipulating flowable substrates like granular media [2] and fluids [3]. We refer to the interactions between self-propelled systems and heterogeneities in the surrounding environment as active collisions. As noted in Ref. [4], conservation of momentum does not apply to collisions in these dissipative and driven systems. As a result, the framework of classical scattering theory is unable to capture the diverse and rich behavior arising from active collisions.

Whether the interactions are among like individuals or between an individual and a heterogeneity, many share a common feature: The driving allows for persistent interactions. These interactions are an important factor in many systems spanning a wide range of length scales, from the aggregation of bacteria near surfaces to form biofilms [5], the self-assembly and disassembly of colloidal clusters [6], and the scattering of spermatozoa and Chlamydomonas from surfaces [7] to locomotion of animals and robots.

Such interactions often hinder movement. For example, Escherichia coli experience a speed reduction near walls [8], self-propelled rods are geometrically captured by cylindrical obstacles [9], and collections of self propelled particles can become jammed in disordered landscapes [10]. In robotic systems, motion planning algorithms often focus on collision avoidance rather than resolution [11]. However, if properly utilized, these interactions can benefit locomotion. For example, rapidly running cockroaches use exoskeletal interactions to maneuver through grass [12] and clutter [13]; sand-specialist snakes experience environmentally induced passive body deformations that enable obstacle negotiation and rapid transit through complex terrain [14]; generalist snakes use body parts to propel from bark and rubble [15]; and Caenorhabditis elegans use environmental structures to enhance propulsion [16]. In robotics, properly tuned dynamical systems can take advantage of periodic mechanical interactions to produce sustained movement [13,17–19] and properly timed tail-ground interactions improve performance.
on yielding substrates [20] and can reduce the effects of collisions [21].

In active systems, interactions and collisions with the environment are persistent, and only when the velocity is directed away from the obstacle or boundary is detachment possible (provided the individual can overcome any other pinning forces and torques). In the microscopic realm, the direction of driving is typically modeled stochastically (arising from Brownian motion) and can include a rotational diffusion term [22]. The strength of the driving and the size of the orientational variations dictate the duration of the interaction as well as the outcome. While the mechanisms by which the orientation can change are different in macroscopic systems, typically either induced by environmental interactions or inherent in the self-propulsion, the ability to reorient remains important for breaking contact with and maneuvering through obstacles. A recent study of environmentally induced passive reorientation found that a robotic cockroach was more successful in traversing narrow openings when biologically inspired body vibrations were added [23].

We study a system in which the orientation is inherent in the self-propulsion of a long slender locomotor that uses undulatory propulsion, in which body bends originate at the head and are subsequently passed down the body to generate movement [24]. This mode of locomotion is observed over a broad range of length scales and produces effective movement in a wide range of environments, from swimming in fluids (e.g., spermatozoa [25], nematodes [26], and aquatic vertebrates [27,28]) to slithering on and within granular materials (e.g., nematodes [29], lizards [30], and snakes [31]) to traversing complex environments (e.g., nematodes [16,32,33] and snakes [15,34]). In particular, we will focus on lateral undulation, in which body bends only occur in the horizontal plane. Despite this restriction, this form of propulsion is still quite general, being the only mode of locomotion shared by all limbless terrestrial vertebrates [35]).

Here we take a robophysics [36] approach, building on our previous work [37] to explore the nature of the interactions underlying active collisions occurring during undulatory self-propulsion in dissipative environments. Details of our experimental and numerical systems are described in Sec. II. Section III A examines interactions with a single obstacle and shows that collisions rotate the robot’s trajectory; Sec. III B demonstrates that this rotation is dominated by head-obstacle collisions. These results motivate the model presented in Sec. III C, which recovers scattering dynamics by reducing the robot to a circular particle and prescribing new collision rules. Section III D uses insights from the model to define a natural collision state space. Section III E extends our investigation to interactions with multiple posts. Surprisingly, scattering patterns produced by a row of evenly spaced posts are reminiscent of far-field diffraction. Section III F shows that multiple-post interactions are still dominated by a single head-obstacle interaction, and Sec. III G shows that collision states are unaltered by the presence of multiple posts, revealing that scattering patterns are generated by altering the distribution of collision states. We close in Sec. IV with a summary of our results as well as a discussion of our work and future studies in the context of driven systems.

II. METHODS

To gain physical insight into active collisions during undulatory self-propulsion, we adopted a robophysical approach and created a laterally undulating sensory-deprived robotic snake [Fig. 1(a)]. Thirteen segments were connected together by N = 12 servo motors, each of which was oriented so that actuation controlled the angular position, \( \zeta_i \), of each motor, \( i \), to vary sinusoidally in time: \( \zeta_i(t) = \zeta_{\text{max}} \sin(2\pi i/N - 2\pi ft) \),

\[ \begin{align*}
F_{\parallel\text{wheel}} & \\
F_{\parallel\text{granular}} & \\
F_{\perp\text{viscous}} & \\
F_{\perp\text{granular}} & \\
F_{\perp\text{headtail}} & \\
\end{align*} \]

FIG. 1. Robophysical snake movement. (a) Robotic snake constructed from servo motors and three-dimensional-printed brackets. Passive wheels affixed to the bottom of each segment enabled the robot to translate, and markers atop each motor were used to track segment positions over time. (b) A schematic of three adjacent servo motors (black circles). The angular position of each motor, \( \zeta_i \), was driven as a function of time. (c) Experimentally measured steady-state wheel forces along (green) and perpendicular to (blue) the rolling direction. Error bars indicate variation over five trials and curves show fits to data. For comparison, drag forces are shown for 300-\( \mu \text{m} \) glass particles (dashed black lines) and for viscous fluid (light gray lines). Forces are scaled so that all \( F_i \) curves have the same maximum value. (d) Robot configurations while moving in a post-free environment. The dashed gray curve shows the corresponding center-of-geometry trajectory. (e) A space-time plot shows experimentally measured \( \zeta_i \) for the trajectory shown in (d). Dashed lines highlight the same band of constant positive \( \zeta \) in each undulation cycle.
creating a serpenoid curve [38] [see Fig. 1(b)]. Here \( f = 0.15 \text{ Hz} \) is the frequency of undulation, and \( \zeta_{\text{max}} = 40^\circ \) is the angular amplitude.

Translational motion of the robot was achieved from the motor-angle actuation through a frictional anisotropy, created by affixing a pair of passive wheels (connected by an axle) to the bottom of each robot segment [39], see Fig. 1(a). To estimate the ground reaction forces for a single robot segment, steady-state drag forces were measured in separate experiments (see Ref. [40]) as a single wheel pair with normal force equal to the weight of a robot segment was translated across the experimental substrate at constant speed [see Fig. 1(c)]. For comparison, drag force relations are shown for other dissipative environments: a submerged rod translated through granular material [31] and a slender rod moving through a viscous fluid [41].

A typical low-slip trajectory of the robot resulting from the serpenoid motion and the wheel-ground interaction is shown in Fig. 1(d). Experimentally measured \( \zeta \), throughout this trajectory (determined from the segment positions) are shown in the space-time plot in Fig. 1(e). The head-to-tail wave progression is confirmed by the diagonal stripes, and the consistency of these stripes throughout four undulations shows that the robot motors reliably followed the prescribed motion.

Heterogeneous environments were created by anchoring either one or five rigid, vertical, force-sensitive cylindrical posts to an otherwise-homogeneous substrate [see Fig. 2(a)]. Example trajectories in single and multipost environments are shown in Figs. 2(b) and 2(c). To characterize the interactions between the robot and the post(s), the robot was initialized to always start in the same configuration: the "S" shape shown in Fig. 1(a). The robot was then placed so that its head was within a box of dimension \( L_x \times L_z \), where \( L_x \) is set by either the amplitude of the robot (single post) or the center-to-center distance between posts (multipost) and \( L_z \) is set by the distance traveled by the robot in a single undulation cycle (see Fig. 2). Outside this region, interactions would either be repeated or the snake would always entirely miss the post.

Multibody physics simulations, created with Chrono [42], allowed for parameter variation and provided additional information (such as time-resolved forces on the robot) not available from the experiment. Experimentally validated simulations were created using the physical parameters of the experiment and the wheel friction relations in Fig. 1(c). The resulting simulations were in good agreement with experiments [40].

III. RESULTS AND DISCUSSION

A. Single post: Scattering distribution

We began with a simple heterogeneous terrain: a single vertical post firmly anchored to an otherwise homogeneous substrate [see Fig. 2(a)]—for these experiments, only the central post was present in the arena. We find that collisions with the post rotate the robot’s direction of travel [Fig. 2(b)], and that, given the low-slip interaction with the substrate, the full trajectory is well approximated by the path traced by the head. We therefore used the head trajectory to describe the robot’s dynamics.

To visualize how collisions affect the spatial distribution of trajectories beyond the post, we generated a probability map, i.e., a statistical image in which each pixel value indicates the fraction of trials the corresponding region in space was traversed by the robot. Figures 3(a) and 3(b) reveal that a structured pattern emerges in both experiment and simulation when initial conditions are densely sampled.

Two notable features in these probability maps are quite different from what we would expect from simple collisions in nonactive systems. First, there are periodic excluded regions (“images” of the post) beyond and directly behind the post, which are reminiscent of features in near-field diffraction patterns. Here these excluded regions occur at integer multiples of \( v_0 T \) [the distance traveled in a single undulation cycle, see Fig. 2(b)]. These forbidden regions arise from the low slip trajectory enforced by the wheels and the physical constraint that the robot cannot penetrate or move the post. While excluded regions could arise in nonactive collisions, the structure would be different. For instance, in the predicted
FIG. 3. Single-post scattering patterns and distributions. Probability maps of head trajectories for (a) 481 experimental and (b) 3000 simulated snake-post interactions. Here the color scale indicates fraction of trials passing through each pixel. (c) Scattering angle distributions for both experimental (dark blue curve) and simulated (dashed light blue curve) snake-post interactions with a single post. Inset: The shaded region shows trajectory angles for 104 experiments in which the robot did not interact with the post. The curve shows a normalized Gaussian fit to the data, with mean $\theta_0 = 0.4^{\circ} \pm 0.1^{\circ}$ and standard deviation $\sigma_\theta = 2.4^{\circ} \pm 0.1^{\circ}$. Uncertainty in fit parameters indicate 95% confidence intervals. To estimate the effect of the experimental error in robot placement would have on the simulation distribution, a noise value was drawn from the Gaussian fit and added to each simulation scattering angle. This process was repeated 10 000 times, and the resulting simulation distribution is the shaded blue region.

scattering pattern of a ball initially traveling along a straight path toward a fixed obstacle, momentum-conserving final trajectories would either miss the obstacle completely or bounce off and scatter backward. In contrast, we find that no single robot trial results in back-scattering or reflection; in all cases, the internal driving ensures that the robot continues to travel forward after the interaction.

Second, we find that most interactions produce small deflections, which we quantify by measuring the scattering angle, $\theta$ [defined in Fig. 2(b)], of all trajectories. Distributions of $\theta$ values for both experiments and simulations have a central peak which confirms that $\theta = 0^\circ$ is the most probable deflection [see Fig. 3(c)]. Distributions show that $\theta = 0^\circ$ occurs more often in the simulation; however, we find that agreement can be improved by accounting for experimental error associated with manual initial placement of the robot at the beginning of each experiment [see inset of Fig. 3(c)].

When noise representative of this experimental error is added to the simulated scattering angles, we recover a simulation distribution in good agreement with that of the experiment.

B. Single post: Individual collisions

To understand how robot-post interactions contribute to the scattering distribution, we used simulations to investigate individual collisions. Robot-post interactions were identified by nonzero contact forces on robot body segments. Approximately half of all simulations have no interaction, $\sim 40\%$ experienced a head-post collision, and the remaining $\sim 10\%$ had only nonhead interactions [see Figs. 4(a)–4(c)]. Unlike collisions in nonactive systems, scattering distributions that only include robot-post interactions remain strongly peaked around no deflection. Additionally, nearly all simulations with a head-post interaction were repulsive (robot deflected away from the post), and simulations with only nonhead interactions were attractive (robot deflected inward toward the post), analogous to Ref. [37].

Given that most interactions included a head-post collision and that the low-slip trajectory ensures that body segments follow the path traced by the head of the robot, we focused on head-post collisions and used the head trajectory to describe interaction dynamics. To quantify the persistence of this interaction, we define the head-post contact duration, $\tau = t_f - t_0$, where $t_0$ and $t_f$ are the initial and final times of contact, respectively. Figure 4(d) shows that scattering angle, $\theta$, varies linearly with $\tau$.
C. Single post: Active collision model

The time evolution of a representative head-post interaction is shown in Fig. 5(a): As the head moves along the post surface, the body-ground contact locally slips and causes a rigid rotation of the entire body. These observations motivate a simple model in which we treat the head as a self-propelled circular particle, illustrated in Fig. 5(b). We prescribe a time-dependent driving velocity, \( \vec{v}(t) \), that matches the unobstructed head velocity. Particle-post contact is established upon initial overlap and is maintained until the particle velocity vector no longer has a component which points in toward the center of the post (i.e., contact ends when \( \hat{v} \cdot \hat{r} > 0 \), where \( \hat{r} \) points radially outward from the post center). During this contact, the particle incrementally moves along the post surface by an amount

\[
\Delta s(t) = c_1 v_\parallel(t) \Delta t,
\]

where \( v_\parallel(t) \) is the component of \( \vec{v}(t) \) that is locally tangent to the post surface at time \( t \) and \( c_1 \) is a constant that should be related to the post-particle friction coefficient (here we find good model-simulation agreement for \( c_1 = 1 \)). While we did not vary the friction coefficient, we expect that \( c_1 \) should be inversely related to the head-post static friction, \( \mu_s \); for \( \mu_s \to \infty \), \( c_1 \to 0 \) and for \( \mu_s \to 0 \), \( c_1 \to 1 \). In simulations presented here, \( \mu_s = 0 \).

As the particle advances along the post surface, the rigid and impenetrable post requires that the velocity component perpendicular to the post must be 0. However, there is a nonzero component to the driving force which would have advanced the particle by an amount

\[
\Delta s_\perp(t) = v_\perp(t) \Delta t
\]

in a homogeneous environment. Given that this motion cannot occur, we suppose that the post effectively pushed the freely moving particle by a distance \( -\Delta s_\perp \) to maintain its location at the post surface and that this push resulted in an infinitesimal rotation,

\[
\Delta \theta(t) \sim -\Delta s_\perp(t)/r_{\text{eff}},
\]

about a point some distance \( r_{\text{eff}} \) away from the contact point. If \( r_{\text{eff}} \) is constant, then the model-predicted scattering angle, \( \theta_m \), is determined by summing incremental \( \Delta s_\perp(t) \) over the duration of the contact.

Figure 5(e) compares the model-predicted and simulation contact durations. Data are clustered around the \( \omega t_m = \omega t_s \) line, showing that the model predicts the duration in most cases. Discrepancies are caused by the model requirement that \( \vec{v}_1 \cdot \hat{z} \geq 0 \) (i.e., the particle cannot travel backward along the surface). While this rule produces good agreement between the model and the simulation in most cases, we find that there is a small region at the leading tip of the post in which this rule does not accurately describe the dynamics. In these cases, the simulated snake head can have a small negative \( v_z \), allowing it to slide backward and around the leading tip of the post, and resulting in a quickly broken contact. In the model, however, the particle is pinned at the same location until the velocity reorients and the particle slides along the post in the \( +\hat{z} \) direction. We find that these points always produce the largest discrepancies between the model and simulation. We expect that a better rule to describe these interactions would improve the agreement between the simulation in this region.

To predict a scattering angle from the model, we assume that \( r_{\text{eff}} \) is constant and related to \( \lambda_{\text{snake}} \), the wavelength of the snake, and we combine Eqs. (2) and (3) to obtain

\[
\theta_m = \frac{c_2}{\lambda_{\text{snake}}} \int_{t_0}^{t_f} -v_\perp(t) \, dt,
\]

where \( \theta_m \) is the model-predicted scattering angle and \( c_2 \) is a constant of proportionality. To estimate the value of \( c_2 \), we compared the right-hand side of Eq. (4) to simulation scattering angles, \( \theta_s \). We find that \( c_2 \approx 1 \) produces a good match. Figure 5(d) compares the resulting model-predicted \( \theta_m \) values with the corresponding \( \theta_s \) values. Most data cluster around the \( \theta_m = \theta_s \) line, indicating that the simplified picture provided by the model is a reasonable approximation for the observed dynamics of most collisions.

Next, we compare the predicted dependence of the scattering angle on the contact duration in Fig. 5(e). The model
captures the observed trend: \( \theta \) varies linearly with \( r \). Given the density of the raw data, we divided the data into windows based on \( \omega \tau \) values and averaged all \( \theta \) values within each bin. These averaged values are in good agreement.

To test the generality of Eq. (4), we ran simulations for different snake shapes. By varying the angular amplitude of the motor oscillations, \( \xi_{\text{max}} \), we change both the amplitude of the overall wave shape as well as the wavelength of the snake. Using the model, we again determine the right-hand side of Eq. (4) and compare with the scattering angles measured from the simulation. We find that \( c_2 \approx 1 \) produces good agreement across amplitude variations investigated here. To demonstrate this agreement, we again compute the average predicted and simulated scattering angles within \( \omega \tau \) bins, as in Fig. 5(e). Figure 6 shows these average scattering angle values plotted against each other. All values fall close to the line \( \theta_m = \theta_s \), indicating that the model is valid over the range of amplitudes explored here.

We next explore how the dynamics depend on physical parameters not varied in the simulation. We therefore investigate how variation in head-post friction, \( \mu_s \), head size, \( r_{\text{head}} \), post size, \( r_{\text{post}} \), and driving speed, \( v \), affect scattering distributions. From geometry, we know that \( \Delta s_{\parallel} = (r_{\text{head}} + r_{\text{post}}) \Delta \phi \), which, when combined with Eq. (1), along with the fact that \( v_{\parallel} \leq v \) and the expectation that \( c_1 \sim 1/\mu_s \), yields

\[
\Delta \phi \leq \frac{v}{\mu_s (r_{\text{head}} + r_{\text{post}})} \Delta \tau,
\]

where \( \Delta \phi \) is the incremental change in post contact location that occurred over time \( \Delta \tau \). From this relation, the model predicts that if \( \Delta \tau, v, \) and \( r_{\text{head}} + r_{\text{post}} \) are constant, then as \( \mu_s \rightarrow \infty, \Delta \phi \rightarrow 0 \). Given that there are more states for which the velocity orientation can meet or exceed tangency as the particle approaches the trailing tip of the post, this reduction in \( \Delta \phi \) means that the particle may either require more time to reach the trailing edge of the post or, alternatively, require more time to achieve tangency at points closer to the leading tip of the post. In either case, this can lead to longer contact durations, which we know from Fig. 5(e) produce larger deflection angles. Therefore, we expect the distribution of scattering angles to broaden as head-post friction increases. Similarly, if we fix \( \mu_s \) and vary \( r_{\text{head}} + r_{\text{post}} \), then we expect that for a larger diameter post or a larger diameter head, \( \Delta \phi \) will again decrease, which will broaden the distribution of scattering angles. Finally, increasing the driving speed with other parameters held constant could enable the particle to overcome higher friction or larger obstacles.

D. Single post: Collision state space

The simplified picture of the model provides insight into the dynamics of undulatory active collisions. In particular, persistent interactions with obstacles lead to locomotor deflections, which are set by the duration of the head-obstacle contact. From the model, we find that the duration of the contact is set by the initial collision state, which is defined by the initial location on the obstacle and the velocity orientation at impact. Therefore, rather than \( xz \) space, we choose a more natural collision state space. We define the initial contact point by the angular location on the post, \( \phi \), and we choose to define the velocity in terms of a wave phase, \( \eta \), over velocity vector orientation, because the phase uniquely specifies location in the undulation cycle. Given that the primary oscillation direction is transverse to the average heading, which is initially in the \( +z \) direction, we define the phase as \( \eta = \tan^{-1}(\dot{x}/\dot{z}) \). For more information about how points are mapped from \( xz \) space to \( \eta \phi \) space, see Ref. [40].

Figure 7 depicts the physical configuration of the snake and the post for several of these collision states. Accessing the

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**Figure 6.** Snake shape variation. The model-predicted scattering angle, \( \theta_m \), compared with simulation scattering angles, \( \theta_s \). Colors represent different shapes, labeled to the right of the main plot. For each shape, \( \theta_m \) is predicted for each simulation. To make comparison easier, \( \theta_m \) and \( \theta_s \) values shown were averaged within identical \( \omega \tau \) bins. Error bars indicate the standard deviations within each bin.

**Figure 7.** Single-post collision state space. Sketches depicting snake configuration and impact location for single-post collision states. The shaded gray region indicates states that are not allowed because they require the robot to travel through the post to reach the correct configuration.
FIG. 8. Single post collision states and durations. Impact location, $\phi$, and wave phase, $\eta$, colored by the duration of the resulting head-post collision for 1200 simulations (left) and predicted for 85 000 points (right).

States in the shaded gray regions would require the robot to travel through the post; therefore, these states are forbidden. States within the white band are allowed, and the dashed line between the two regions indicates the boundary between allowed and disallowed states.

We use the model to predict $\omega \tau$ for all possible collision states, and we compare with results from simulation in Fig. 8(a). The structure of $\omega \tau$ as a function of $\eta$ and $\phi$ is qualitatively similar: Both are contained within the same region, whose boundaries are identical to those of Fig. 7, and while there are quantitative differences between the simulation and the prediction [40], the dependence of the duration on the collision state is qualitatively similar.

The model provides a framework for describing active collisions in damped-driven systems. Heterogeneities in the environment impose geometric constraints which can prevent active particles from fully utilizing their internal driving to produce movement. The degree to which obstacles hinder locomotion depends on the details of the driving and the shape of the obstacle: For an undulating locomotor interacting with round posts, we find that the duration of the interactions is set by the undulation phase and post impact location at the initial time of contact. The locomotor is “stuck” to and can only move along the surface of the obstacle until the velocity vector reorients and has a component pointing away from the obstacle. We note that, in this picture, the contact duration is qualitatively equivalent to the reorientation time of many other active matter systems (see, e.g., Ref. [22]). However, unlike active Brownian systems and those which experience a purely passive reorientation [23], the reorientation is largely inherent in the driving of the locomotor. For a periodically driven locomotor interacting with a single post, we find that

FIG. 9. Scattering patterns and distributions varying post spacing. Probability maps of robot head trajectories for three post configurations in experiment (left column) and simulation (middle column). Here the color scale indicates fraction of trials passing through each pixel. $d$ is constant across each row and increases down each column. Right column: Experiment (dark blue curves) and simulation (shaded light blue regions) scattering angle distributions for three post spacings, $d = 5.7$ cm, $d = 6.9$ cm, $d = 9.0$ cm, each of which contains at least 300 trials. Dashed vertical lines show the angles associated with the outer $\pm15\%$ of each distribution (i.e., the 15th and 85th quantiles).
collision durations (and corresponding locomotor reorientations) increase as initial impact locations approach the leading edge of the post.

### E. Multiple posts: Scattering distributions

We next explore how the presence of multiple obstacles alters scattering distributions and underlying collision states. Here five evenly spaced vertical posts were firmly anchored to an otherwise homogeneous substrate [a schematic is shown in Fig. 2(a)]. A representative experimental trajectory is shown in Fig. 2(c). Similarly to the single-post interactions, here the robot is rotated by the collisions with the posts. A probability map of multipost trajectories shows the likelihood of the robot to occupy points in space after the collision (Fig. 9). When the initial conditions were densely sampled (shown in the left and central columns), a structured pattern appeared and the presence of preferred trajectories emerged.

![FIG. 10. Distribution spread dependence on post spacing and segment angular amplitude. (a) $\theta_0$ vs. $d$ for experiment (dark diamonds) and simulation (light circles). Numerically labeled points result from corresponding distribution in Fig. 9. Error bars indicate bootstrapping-estimated 95% confidence intervals associated with each quantile measurement. The curve shows the fit of $\theta_0 = 180/\pi(D/d)$ to the simulation, with the shaded region indicating 95% prediction bounds for the fit. (b) $\theta_0$ vs. $1/d$ measured from distributions for different $\ell_{\text{max}}$ (varied in simulation). Lines show fits to the data, and shaded regions indicate 95% prediction bounds for each fit. Corresponding wave shapes are shown to the right. (c) The fit parameter, $D$, for different $\ell_{\text{max}}$. $D$ is linearly related to the full perpendicular distance each segment sweeps out in one period. Inset: schematic of single motor and two adjacent segments. The perpendicular distance swept out by a single segment during a full cycle is given by $2\ell \sin \ell_{\text{max}}$.](image)

![FIG. 11. Scattering angle dependence on head-post contact duration. Scattering angle, $\theta$, depends linearly on $\ell_{\text{max}}$, the longest head-post contact duration, even in the presence of multiple posts. The gray line, determined by fitting the single-peg data in the bottom right, is the same in all plots and shows that this trend is independent of post spacing, $d$. The underlying color scale represents the two-dimensional probability map and shows that the density of points shifts inward along the $\theta$ vs. $\tau$ line as spacing increases. The plot in the bottom right shows the probability map version of the single post data shown in Fig. 4(b).](image)
d and tend to occur at preferred directions. This produces secondary peaks in the scattering distributions which become more prominent as spacing decreases.

F. Multiple posts: Collision durations

Given the importance of the head-post contact duration for the single-post environment, we again explore the relationship between the contact duration of the head with the posts. In the multipost geometry, the head can have multiple collisions which can involve more than one post. However, we find that there is typically one head collision that dominates; therefore, we start by examining the single collision with the maximal head-post contact duration, and we restrict our analysis to simulations which had at least one head-post collision [40].

Figure 11 shows that, even in the multipost configuration, \( \theta \) depends linearly on \( \tau_{\text{max}} \) and that this relationship is independent of \( d \). When each plot is viewed as a probability map, the density of points along this line increases. Given this linear relationship, we expect that the spread of both the \( \tau_{\text{max}} \) and \( \theta \) distributions should exhibit a similar dependence on spacing.

We explore this potential similarity by comparing the qualitative dependence \( \tau_{\text{max}} \) and \( \theta \) quantiles on the spacing. Figure 12(a) shows distributions of \( \tau_{\text{max}} \) for three \( d \). We again choose the 70th quantile to characterize the spread of the distributions. Figure 12(b) shows the qualitatively similar distribution. (c) \( \phi_{\text{NS}} \) as a function of \( d \). Error bars show the bootstrapping-estimated 95% confidence interval for each \( \phi_{\text{NS}} \) value.

We have neglected many details of the interactions that occur along the robot body as it traverses the post array.
and have shown that we can reduce the system to a single interaction: the longest-duration collision. Not only does this indicate that the resulting dynamics are dominated by the longest head-peg interaction, but it also suggests that, at least for our system, only one post is important in a given single- or multipost scattering event. It would be interesting to explore situations (for instance, trajectories with higher slip or a more complex arrangement of obstacles) in which there may be multiple important interactions.

G. Multiple posts: Collision states

To understand how active collisions in the presence of multiple posts can generate the observed scattering patterns, we examine the unobstructed path of the robot. This path is shifted to coincide with an initial condition that results in a collision for both $d = 5.7$ cm as well as for the single post, see Fig. 13(a). From this picture, we see that the single-post collision, which occurs opposite the leading surface of the central post, becomes inaccessible in the multipost scenario. Instead, a collision with the post immediately to the left precedes the single-post interaction. This new collision with an adjacent post occurs closer to the leading surface of the post, which, at least in the single post case, can result in a longer-duration collision.

We expect that as spacing decreases, states near the trailing edge of a post become inaccessible as these trajectories are intercepted by and occur closer to the leading edge of an adjacent post [see Fig. 13(a)]. To test this hypothesis, we quantified how the tail of the $\phi$ distribution depends on post spacing. Distributions for three $d$ are shown in Fig. 13(b), and the dependence of $\phi_{95}$ on $d$ is shown in Fig. 13(c). Not only was the impact location altered by the presence of multiple posts, but it is clear from Fig. 13(a) that the phase of the undulation cycle on impact was also changed. Scatter plots in Fig. 14(a) show how these collision states in $(\eta, \phi)$ space depend on spacing. As $d$ decreases, fewer states are accessible to the robot, and the states that become inaccessible are those away from the leading edge of the post. Aside from this restriction on allowed states, the dependence of $\tau_{\text{max}}$ on $\eta$ and $\phi$ is nearly the same. This suggests that collision states are largely independent of $d$.

To test the similarity of collision states for different post configurations, we compare the single post collision state closest to [i.e., smallest Euclidean distance in the $(\eta, \phi)$ space from] each multipost state in $(\eta, \phi)$ space [40]. If the states are equivalent, then contact durations associated with the single and multipost state should be identical. Figure 14(b) shows the probability maps of three multipost durations as a function of the nearest single-post state. For all three $d$, most of the data fall along the $\omega\tau_{\text{multi}} = \omega\tau_{\text{single}}$ line, confirming that adjacent posts act primarily to shift the probabilities of single-post collision states. As the spacing decreases, single-post states near the trailing edge of the post occur with reduced probability (and some are even eliminated completely) as trajectories are “remapped” to a different single-post collision state occurring at an adjacent post. These shifted collisions tend to occur closer to the leading tip of the post than the...
original collision, often resulting in longer durations than the single-post state that was replaced. Given the linear relationship between duration and scattering angle, the remapping from shorter to longer durations shifts power from the central peak of the $\theta$ distributions out to the tails, creating and bolstering secondary off-center peaks.

To explore how single-post states are shifted by the presence of multiple posts, we identify the multipost point closest to each single post point in $(x, z)$ space. To do this, we tiled the multipost initial conditions box [e.g., for $d = 5.7$ cm, the solid box in Fig. 15(a)] by shifting all points within this region over by $\pm m L_x$, where $m$ is an integer and $L_x$ is the transverse dimension of the initial conditions box. Outlines for shifts of $m = \pm 1$ are shown as the dashed boxes in Fig. 15(a). The points within each box show the starting point for the head of the robot, and the colors indicate which post was involved in the longest-duration collision with the head of the robot. When initial conditions were shifted, a different post was centered in front of the box, and given that all initial conditions boxes are identical, the post number associated with a collision in a box shifted by $m$ post must also be shifted by $m$.

In Fig. 15(a), the multipost points for $d = 5.7$ cm are shown in varying shades of blue, and the single-post points (all of which hit the central post, outlined in black) are overlaid in black. To identify where the single post points were shifted around in $(\eta, \phi)$ space, we determined the $x_z$ distance between each single-post point and the nearest multipost point, $\delta_{xz} = \sqrt{(x_z - x_m)^2 + (z_z - z_m)^2}$, which was rarely larger than 0.5 cm. The colored “x” markers in Fig. 15(a) identify four regions which hit post $n$ in the single-post case but were involved in more significant collisions with adjacent posts in the multipost case. How these regions were shifted around in $(\eta, \phi)$ space is shown in Fig. 15(b). The “x” points were shifted to the circular points of the same color. Figure 15(c) shows nearly all of the remapped points had significantly longer durations, $\tau_{\text{remap}}$, than the original single-post collision, $\tau_{\text{orig}}$. The line shows $\omega \tau_{\text{remap}} = \omega \tau_{\text{orig}}$.

These results confirm that single-post collision states are largely unaltered by the presence of multiple posts, even when $d$ is small. Instead, multiple posts serve to restrict the collision states accessible to the robot. As $d$ decreases, low-duration states occurring near the trailing edge of the posts become inaccessible and are replaced by longer-duration collisions near the leading edge of an adjacent peg. Stated another way, scattering events with small reorientations are preferentially remapped to larger-angle scattering events.

IV. SUMMARY AND CONCLUSIONS

The results presented here provide a striking example of the dynamics that can arise in self-propelled systems when environmental heterogeneities are present. To explore the nature of the interactions that can occur during undulatory self-propulsion, we created a robophysical snakelike robot which self-deforms by propagating a wave of joint-angle variations from head to tail. Passive wheels enable the robot to translate by creating a highly dissipative coupling between these self-deformations and the surrounding environment. We find that the nature of this dissipation is similar to that of both viscous fluids (relevant for swimmers in low Reynolds number fluids) and granular materials (relevant for movement on and within sand). This suggests that our results may be relevant to biological and artificial systems spanning a broad range of length scales and environments.

Interactions with a single obstacle (a rigid vertical post) scatter the robot, and, unlike simple collisions in nonactive systems, the distribution of scattering angles produced by interactions with a single post is strongly peaked directly behind the post. When multiple posts are present, secondary
peaks emerge and become more prominent as post density increases, producing scattering patterns reminiscent of far-field diffraction. Surprisingly, we find that the collisions are not altered by the presence of multiple posts; instead, the likelihood of collisions shifts so that there are more interactions which produce large-scattering events. In all cases, the resulting scattering angle is proportional to the head-post collision duration. A simple model reveals that this duration is similar to the reorientation times discussed.

Simulations allowed for broader parameter variation and revealed that, like movement through similarly highly dissipative environments, our results are independent of the frequency of undulation. That is, the linear relationship between the head-post contact duration, $\omega \tau_{\text{max}}$, and the resulting scattering angle remains the same for a broad range of frequencies. Variation of the angular amplitude, $\zeta_{\text{max}}$, also did not significantly alter the linear dependence between the duration and the scattering angle, suggesting that our results are valid for a range of waveforms and undulation frequencies. Our model, which only considers the head of the snake, confirms this and is able to predict contact durations over the range of amplitudes investigated. With the addition of a lever arm (i.e., an effective length about which the snake is rotated), the model predicts scattering angles over the range of amplitudes studied. Surprisingly, simulation and model-predicted scattering angles agree when the lever arm length is equal to the wavelength of the snake.

Broadly speaking, our results provide a new approach to modeling collisions in active systems within dissipative environments, which could be applicable to both deterministically and stochastically driven reorientations. In at least some situations, such as the system presented here, self-propelled agents can be reduced to driven particles which possess effective physical properties and follow simple collision rules that are set by geometric constraints of the environment. Extending our model to create a general framework which draws inspiration from systems with both particle-like characteristics and effective wave-like properties may provide insight into a wide range of periodically driven systems. In particular, it would be interesting to explore the potential connections to other systems which produce similar scattering patterns, such as biological snakes in heterogeneous environments [14] and bouncing fluid droplets interacting with substrate disturbances that they create [44,45].

We close by noting that robophysics provides a useful approach for exploring the nature of active collisions across scales and environments because it enables controlled experiments and systematic parameter variation while avoiding the complexities and unknowns of numerical collision-modeling and the variability and controllability difficulties found in living systems. Robophysics is widely applicable and amenable to other modes of locomotion, body morphologies, and obstacle configurations and geometries. With an understanding of active collisions, these interactions could be used to mitigate or even utilize interactions with heterogeneities for different classes and environments for natural and artificial locomotors, e.g., in legged [12,46], undulatory [33,47], sidewinding [48], wheeled and tracked vehicles [49,50], and even aerial systems [51,52]. Alternatively, environments could be designed to direct the motion of self-propelled systems, for instance, to correct for (e.g., Ref. [7]) or selectively enhance scattering effects. Finally, structured environments could also be used to modify the duration of these interactions, which, given the importance of the interaction duration on the dynamics of active systems, could have broad implications for collective behavior in biological and artificial systems.

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[43] We note the small angle approximation ($\sin \theta \approx \theta$) is valid for angles we measure, so a fit to the function expected for far-field wave diffraction, $\theta = 180/\pi \sin^{-1}(D/d)$, is indistinguishable from the fit we have chosen.


