Emergence of the advancing neuromechanical phase in a resistive force dominated medium

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Undulatory locomotion, a gait in which thrust is produced in the opposite direction of a traveling wave of body bending, is a common mode of propulsion used by animals in fluids, on land, and even within sand. As such, it has been an excellent system for discovery of neuromechanical principles of movement. In nearly all animals studied, the wave of muscle activation progresses faster than the wave of body bending, leading to an advancing phase of activation relative to the curvature toward the tail. This is referred to as “neuromechanical phase lags” (NPL). Several multiparameter neuromechanical models have reproduced this phenomenon, but due to model complexity, the origin of the NPL has proved difficult to identify. Here, we use perhaps the simplest model of undulatory swimming to predict the NPL accurately during sand-swimming by the sandfish lizard, with no fitting parameters. The sinusoidal wave used in sandfish locomotion, the friction-dominated and nonlinear granular resistive force environment, and the simplicity of the model allow detailed analysis, and reveal the fundamental mechanism responsible for the phenomenon: the combination of synchronized torques from distant points on the body and local traveling torques. This general mechanism should help explain the NPL in organisms in other environments; we therefore propose that sand-swimming could be an excellent system with which to generate and test other neuromechanical models of movement quantitatively. Such a system can also provide guidance for the design and control of robotic undulatory locomotors in complex environments.

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Model

Resistive Force Theory Model. Previously, we developed a granular resistive force theory (RFT) model and a numerical simulation that explained the swimming performance of the sandfish (18, 34, 35). The models showed that the lizards swim within a self-generated “frictional fluid,” wherein frictional forces between the granular particles dominate both body inertia and inertial forces from the environment. As before, we prescribe body kinematics (in the frame of the animal) based on the experimental observation that the body position of a sandfish in the body frame is approximately a single-period sinusoidal wave traveling posteriorly (Fig. 2):

$$y_b = A \sin \left[ 2\pi \left( \frac{x_b}{\lambda} + \frac{t}{T} \right) \right],$$

where $y_b$ is the lateral displacement from the midline of a straight animal, $A$ is the amplitude, $T$ is the period of undulation, $\lambda$ is the wavelength, $t$ is the time, and $x_b$ is the distance along a line parallel to the direction of the traveling wave measured from the tail tip. Here, we normalize both the wavelength and period to $2\pi$ such that $\lambda/2\pi = 1$. Because the trunk of the sandfish is quite uniform [with body width variations less than about 5% from 0.1 snout-to-vent length (SVL) to 1.0 SVL] and the diameter of the body decreases significantly after about 1.2 SVL, we used a uniform body shape and took the total arc length ($L$) in the model to be 1.2 times the average SVL (8.9 ± 0.3 cm) of the animal. Dissection revealed after 1.2 SVL, the tail is composed of mostly adipose tissue and a small amount of muscle; therefore, both the external and internal torques on the tail should be minimal for the tail beyond 1.2 SVL. We neglected the variation of the horizontal position $x_b$ of a segment within a cycle; thus, the normalized position on the animal body, $\sigma = x_b/\lambda$, corresponded to the horizontal position $x_b$ in the model, where $\sigma$ is the arc length from the tail end. When a smaller amplitude was used, the wavelength was kept as $2\pi$.

For swimming in sand, the granular force, $F$, on any infinitesimal segment of the swimmer is independent of the segment speed (and thus undulation frequency), proportional to its depth, and is a function of the angle ($\psi$ in Fig. 2) between the segment axis and its velocity direction (the empirically determined granular force, $F(\psi)$, is shown in Supporting Information). The depth of a segment is calculated assuming the model sandfish swims with its center 3.5 cm below the horizontal plane and at an entry angle of 22° [an average value for the sandfish (18)]. The entry angle is the angle between the horizontal plane and the plane in which the animal moves (26).

Because the estimated inertial force is negligible, the swimmer moves in a way such that net external force and torque are approximately zero. In this study, we consider all 3 df in a plane, namely, the forward (the only df in our previous RFT models), lateral, and yaw motion (“recoil”), and we determine the velocities of the 3 df by solving the force/torque balance equations at every instant of time. Because the motion of a point on the body is the superposition of the prescribed and center of mass (CoM) motions, the net external force, $F_{net}(\dot{R}, \theta)$, and net external torque about the CoM, $\tau_{net}(\dot{R}, \theta) = (0, 0, \tau_{net})$, are functions of the
CoM velocity \( \dot{\mathbf{R}} \) and rotation rate about the CoM \( \dot{\theta} \). For the CoM movement, Newton’s laws give

\[
\begin{align*}
\dot{\mathbf{F}}_{\text{net}}(\mathbf{R}, \dot{\theta}) &= M\ddot{\mathbf{R}} \\
\tau_{\text{net}}(\mathbf{R}, \dot{\theta}) &= \dot{L},
\end{align*}
\]

where \( M \) is total mass and \( L \) is angular momentum. By setting the inertial terms on the right sides of these equations to zero, the CoM velocities (\( \dot{\mathbf{R}} \) and \( \dot{\theta} \)) can be numerically determined.

**Torque Calculation in RFT.** Because inertia is negligible, the net torque due to the granular force on a portion (e.g., \( [x_0, 2\pi] \)) of the sandfish body about any point of interest \( x_0 \) is also approximately zero. From this, we calculate the internal torque (i.e., the torque generated by muscle) at \( x_0 \):

\[
\tau_{\text{muscle}} \equiv \int_{x_0}^{2\pi} \mathbf{f}_g(x, t) \cdot \mathbf{dx} = \dot{L}(t) \approx 0
\]

\[
\tau_{\text{muscle}} = \int_{x_0}^{2\pi} \mathbf{f}_g(x, t) \cdot \mathbf{dx}
\]

where \( \mathbf{f}_g \) is the granular force per unit length. We assume the muscle must only overcome torque from resistive forces \( \tau \), thus, internal passive body forces are small compared with external resistive forces. This assumption was tested by performing in vivo bending tests on an anesthetized animal (Materials and Methods and Supporting Information) and measuring stiffness and damping coefficients at varying rotation rates; we estimate that the maximal torques from elastic \((0.094 \pm 0.027 \text{ N-cm})\) and damping \((0.055 \pm 0.034 \text{ N-cm})\) forces are over an order of magnitude smaller than the maximal torque from resistive forces \((4.1 \text{ N-cm})\). We also assume the time lag between neural activation and muscle force development is small compared with the sandfish undulation period \((\approx 0.5 \text{ s})\). We thus assume activation timing approximately corresponds to “muscle” torque timing. Therefore, we use the sign of \( \tau \) to predict muscle activation (Figs. 2 and 3): Positive \( \tau \) (or negative \( \tau_{\text{muscle}} \)) corresponds to muscle activation on the right side of the body.

**Results and Discussion**

We find that phase lags between internal torque and curvature in the model can explain the NPL between the electromyogram (EMG) and curvature seen in experiments. A traveling wave pattern is displayed by \( \tau \), and positive \( \tau \) occurs in a range close to that of measured EMG activation (Figs. 3 and 4). Without corrections from body passive forces or consideration of muscle physiology or body structure, the average phase difference between the beginning and ending of positive \( \tau \) in the model compared with EMG onset and offset in experiments is less than 5%, where \( 2\pi \) is the range of possible phase lags (Supporting Information). A large portion of the positive torque region overlaps with the region in which the curvature decreases (negative \( \kappa \)), but the positive \( \tau \) region lags behind the negative \( \kappa \) region near the head and leads it near the tail. The agreement between experiment and theory is striking, particularly because our model has no fitting parameters; we posit this is largely a consequence of the simple movement and the relatively simple but strong environmental interaction.

To gain more insight into how the phase lags arise due to torque contributions from different parts of the body, we consider a simplified case in which amplitude is small, forward motion is negligible, and the resistive force is viscous. This makes analytical calculation of torque straightforward but does not change the results qualitatively. In this simpler case, the torque from the fore-aft forces is negligible, and only the lateral force (per unit length), \( f_x(x, t) = -c\dot{y}(x, t) = -c\dot{y}\cos(x + t) \), need be considered.

For simplicity and to separate the effects, we first neglect yaw motion. The torque can be calculated analytically from Eq. 3: \( \tau(x_0, t) = (2\pi - x_0)\sin(t) - \cos(x_0 + t) + \cos(t) \). For example, if we take two points \( x_1 = \pi \) and \( x_2 = \pi - \Delta \) near the middle of the body, we obtain \( \tau_1 = 3.7 A \cos(t + \phi) \) and \( \tau_2 = (3.7 + 1.7\Delta) A \cos(t + \phi - 0.29\Delta) \), where \( \phi = 0.57 \). The NPL are still captured because the phase difference between \( \tau_2 \) and \( \tau_1 \)
is a fraction (0.29) of \( \Delta \), the phase difference between \( k_1 \) and \( k_2 \). The torque contribution can be approximately divided into three parts, as follows:

\[
\tau_1 = \int_{x_1}^{2\pi} f_1(z,t)(z-x_1) \, dz \\
\approx \delta f_1(x_1,t) \delta / 2 + \int_{x_1+\delta}^{2\pi-\delta} f_1(z,t)(z-x_1) \, dz \\
+ \delta f_1(2\pi,t)(2\pi-x_1) \tag{4}
\]

where \( \delta \) is a small length. The phase difference between the torque contributions from local forces for the two points is \( \Delta \), which is the same as the phase difference of other local variables (e.g., \( x \)) on the traveling wave (Fig. 5). In contrast, the phase of the torque transmitted from a distant point on the body (e.g., the head) is the same for both points (even though the magnitude differs). This synchronized torque contribution can be thought of as either a standing wave or a traveling wave with infinite speed. Because of the combination of the torques from local and distant forces and the continuous force distribution, the net phase difference between \( \tau_2 \) and \( \tau_1 \) is less than \( \Delta \) and the torque wave speed is greater than the curvature wave speed. A similar analysis can be performed if the integration is done on the posterior side of the body (toward the tail).

The balance of torque on the body leads to an overall yaw motion, whose phase is the same along the body. Superposition of yaw motion and lateral motion of the body results in variation of both the magnitude and phase of the lateral motion along the body in the laboratory frame (See figure and derivation in SI Text). However, the overall speed of the lateral displacement in the laboratory frame is the same as the prescribed lateral displacement (sinusoidal wave) in the body frame. Therefore, the yaw motion only changes the relative phase between the curvature wave and the apparent displacement (or force) wave locally.

Because the only requirement for this mechanism is a traveling wave pattern of force, it predicts the NPL are general for torques from distributed forces. As shown in Fig. 6A, localized elastic and damping forces, by definition, have constant phase differences with the curvature. In accord with previous studies (28, 30), our calculations show that the relative phase between the torque from inertial forces and curvature advances in the posterior direction. However, the overall phase of the inertial torque advances by about 0.4\( \pi \) compared with the sandfish EMG phase. The phase lags persist if the granular resistive forces in the model are replaced with viscous resistive forces, which low Reynolds number swimmers like nematodes experience (13).

Although passive body forces are not responsible for the NPL, they can still influence the observed pattern. For example, we find that the inclusion of viscous forces in the body shifts the phase of the torque in granular media toward the phase pattern produced from only viscous forces (dashed-dotted red lines in Fig. 6A) that is, the phase difference between the torque and \( \dot{x} \) is smaller and the torque wave speed is smaller, in accord with previous studies in fluids (10, 29). This suggests that the small internal viscous forces within the body may partially account for the phase differences we observe between the torque from resistive forces and EMG. For swimming in a fluid with a high Reynolds number, the duration of muscle activation is generally smaller than that observed for the sandfish (\( \approx 0.5 \text{s} \)) (23). Previous studies (e.g., ref. 29) suggest that the torque from external forces may be overcome by passive elements of the body. The nearly 0.5 duty factor of the muscle is evidence that resistive forces dominate in a granular environment, and the slight decrease of the duty factor [a relatively larger decrease is typical during swimming in fluids (23)] implies passive forces play a small role in swimming of the sandfish.

Variations of locomotor kinematics also affect the timing of the torque (Fig. 6B). For example, a downward entry angle [observed in the animal experiments (26)] advances the phase of the torque compared with the horizontal swimming case. This occurs because when the body is oriented downward, the head, which has a more advanced phase, contributes more to the overall torque due to its greater depth and correspondingly larger resistive forces. Also, a larger number of periods (longer body and smaller ratio of wave length to body length) both delays the phase of torque and reduces the torque wave speed. The phase shift is due to the contribution of the extra tail length, where the phase of the
force lags behind that at anterior positions. The effect of period (body length) can be used to estimate the error in timing that may occur due to neglecting the tail after 1.2 SVL. The error should be a small fraction of the difference between the 1.2-period case and the control case. Further, we found that a smaller undulation amplitude reduced the variation in torque wave speed.

The time delay between EMG activation and force production (36) might affect the phase lag timing of EMG activation, but we argue that this delay is small compared with the typical period of undulation for the sandfish (∼2-ms latency compared with ≈500-ms undulation period). If the time delay were significant and approximately constant, the EMG/curvature phase relation would change for different frequencies.

**Conclusions**

We developed a theory to explain the basic control signals needed to generate a particular undulatory movement pattern in a sand-swimming lizard. We abstracted the nervous/musculoskeletal system by assuming that passive body forces are small and that internal torque is synchronized with neural activation timing; this abstraction revealed that the NPL are intrinsic to undulatory locomotion, provided that distributed forces, such as resistive or inertial forces, play major roles. For undulatory locomotion in other environments, the principle of the simultaneous response to distant torques should also apply, although quantities, such as the phase of the force, will differ from the sandfish case. Building on this principle could help future studies explain other variations of the NFL.

Because we now have a system in which experiment and theory are in quantitative agreement, we can begin to develop more detailed models [i.e., anchors (27)] that answer specific questions about nervous system control, muscle configuration, and morphology, for example. As a case in point, it has been established that the intersegmental coordination of neural oscillators along the body of swimmers is influenced by sensory feedback (37). Detailed models of central pattern generators, sensory neurons, and muscles can be used to understand how external torque and neural activation interact so that the intersegmental phase lags produce single-period sinusoidal motion. As such, a hierarchy of anchors can be used to generate testable hypotheses and understand actuation timing for animals in a variety of environments.

More broadly, we have demonstrated that the seemingly specific and peculiar sand-swimming behavior could be an excellent system in which to develop quantitative models of neuromechanics. Due to relatively simple but dominant environmental interactions, the neuromechanical control pattern is greatly constrained by the environment. In addition, the granular RFT provides an excellent model for interaction with the substrate; this is in contrast to locomotion in true fluids, in which more complex theories (38) are needed to compare experiments and models quantitatively. We hypothesize that by studying subarenaceous animals within dry and saturated granular substrates (e.g., those on the bottom of the ocean floor), animal models with potentially fewer parameters can be analyzed in detail. This, in turn, can help provide guidance for the design and control of artificial undulatory locomotors in complex environments (39–41). Better physical models can also improve our understanding of the biological systems.

**Materials and Methods**

**EMG Recordings.** Previous work (26) using a micro-CT scan of a single sandfish revealed 26 vertebrae in the trunk and more than 13 anterior caudal vertebrae in the tail. The illoco costalis musculature was targeted for implantation and is located on the dorsolateral portion of the trunk. Dissection revealed quantitatively similar muscle morphology to that described for Iguana iguana (42, 43), where illoco costalis musculature spanned ~1 vertebrae.

Electrodes were implanted in one side of the body at 0.3 (magenta), 0.5 (green), 0.7 (blue), and 0.9 (yellow) SVLs (Fig. 1B), where the average SVL was 8.9 cm (n = 5 animals). EMG data used in this paper were taken from sandfish swimming trials (n = 37). The EMG signal was filtered with a second-order Chebyshev filter and rectified to facilitate EMG burst detection. A burst threshold was set equal to the mean of this rectified filtered EMG trace. Burst onset was defined as the time when the filtered EMG signal exceeded the threshold and afterward remained above it for a minimum of 0.04 s. EMG burst offset was defined as when the filtered EMG signal became lower than the threshold and remained below it for at least 0.08 s (44). This burst detection was necessary to exclude small voltage changes that did not constitute an EMG burst, such as noise due to movement artifact. More details on the EMG recording and analysis technique are provided elsewhere (26).

**Dynamic Bending Tests.** Three anesthetized sandfish (mass = 15, 16, and 25 g) were gently clamped at ~0.5 SVL and 0.6 SVL (Fig. 7A) with adjustable grips. The grips were attached to a rigid platform and to a rotating platform, respectively. A motor rotated the anterior region of the sandfish through ±15° for three cycles at angular velocities of 1, 10, and 20°/s. The first and last half-cycles were excluded from the analysis due to varying rotation velocities. The anterior end of the sandfish was clamped to a platform with two strain gages (KFG-3-120-C1-11L1M2R; Omega) used to record resulting torques. Signals were amplified (INA125P; Digi-Key) by 5,000 before data acquisition and analyzed using custom software (LabVIEW; National Instruments). Black points were

![Diagram](image-url)
marked on the animal midline at increments of 0.1 SVL. The best-fit line through the markers circled in red was used to calculate the angle \( \theta \).

In the model, we determined the moment of the sand fish, which was a vector \( \mathbf{r}_{\text{C}} \) that was approximated by the viscous torque \( \tau_v \) at zero angular displacement during steady-state rotation and dividing by the angular velocity \( \omega \), i.e., \( \tau_v / \omega \).

For the hysteretic damping model, the structural damping coefficient, \( h \), was proportional to angular displacement \( \theta \) and to \( \pi / \omega \)-out-phase. The value of \( h \) was estimated by finding the torque at zero displacement during steady-state rotation and dividing by the maximum angular displacement: \( h = \tau_v(\theta = 0) / \omega \). The loss factor was calculated as \( h / \omega \), and the value within the work loop \( (\tau_v, \theta) \) was determined using polynomial fits to the torque vs. angle curves for increasing and decreasing angles.

To interrogate stiffness and damping coefficients at higher speeds, we repeated the experiment with one of the sand fish, and we used a viscoelastic model (or Voigt model), the viscous damping coefficient, \( c \), was approximated by evaluating the viscous torque \( c \theta_v \) at zero angular displacement during steady-state rotation and dividing by the angular velocity \( \omega \), i.e., \( c = \tau_v(\theta = 0) / \omega \).

For the hysteretic damping model, the structural damping coefficient, \( h \), was proportional to angular displacement \( \theta \) and to \( \pi / \omega \)-out-phase. The value of \( h \) was estimated by finding the torque at zero displacement during steady-state rotation and dividing by the maximum angular displacement: \( h = \tau_v(\theta = 0) / \omega \). The loss factor was calculated as \( h / \omega \), and the value within the work loop \( (\tau_v, \theta) \) was determined using polynomial fits to the torque vs. angle curves for increasing and decreasing angles.

The same sand fish were used as in the previous experiment. Animals were oriented vertically and clamped at \( -0.5 \) SVL. The tail of the sand fish was bent upward and released, allowing the body to swing freely. The sand fish body was modeled as a rigid cylinder, and the tail was modeled as a cone with uniform density. The angular motion, \( \theta \), was fit to a damped harmonic oscillator:

\[
\ddot{\theta} + \omega_i \theta + \frac{mgd_{\text{cyl}}}{\omega_i^2} \sin(\theta) = 0,
\]

where \( d_{\text{cyl}} \) is the distance from the point of rotation to the center of mass, \( m \) is the mass of the undisturbed portion of the sand fish, \( i \) is the moment of inertia, and \( \omega_i \) is the angular acceleration. Angular motion during the first half-cycle after the tail was released was neglected due to large angles and body bending. The value of \( \omega_i \) was determined between the 0.5-SVL and 0.8-SVL body positions.

We also fit the motion using a hysteretic damping model:

\[
\ddot{\theta} + (1 + h) \omega_i \theta + \frac{mgd_{\text{cyl}}}{\omega_i^2} \sin(\theta) = 0.
\]

For both models, we used the small-angle approximation \( \sin(\theta) \approx \theta \). Best-fit parameters were determined using minimization techniques (MATLAB; MathWorks). Both viscous and hysteretic models fit the angular displacement trajectory well \( (r^2 < 0.9) \). Experimental setup diagrams and detailed results are provided in Supporting Information.

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Supporting Information

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SI Text

**Empirical Granular Force Laws in the Resistive Force Theory**

To obtain the empirical force laws for the forces acting on the sandfish, we dragged a stainless-steel cylinder (radius \(r = 1.58 \text{ cm}\) and length \(l = 4 \text{ cm}\)) in 0.3-mm glass beads at a constant velocity (10 cm/s) and measured the resulting perpendicular and parallel forces for angles \(\phi\) between the element and its displacement direction. The perpendicular \(F_{\perp}\) and parallel \(F_{||}\) components of the force on the cylindrical surface of the rod as a function of \(\phi\) are approximated as

\[
F_{\perp} = 2r(C_s \sin \beta_0 + C_f \sin \phi),
F_{||} = 2rC_f \cos \phi,
\]

where \(\tan \beta_0 = \cot \gamma_0 \sin \phi\), \(C_s = 0.51 \text{N/m}^2 \times 10^{-4}\), \(C_f = 0.28 \text{N/m}^2 \times 10^{-4}\), and \(\gamma_0 = 13.84^\circ\). Details of the empirical force laws are provided in Maladen et al. (1).

**Small-Amplitude Swimming in a Viscous Fluid**

For the small (infinitesimal) amplitude, \(A, \phi\) case, \(\Delta \text{CoM} \approx 0\) because previous studies (e.g., ref. 2) showed \(\Delta \text{CoM}\approx A^2\) for swimming in a viscous fluid. Due to the symmetry of a sinusoidal wave in the lateral direction, \(F_{\text{net,y}} = 0\) and \(\gamma_{\text{CoM}} = 0\). The contribution to forward motion from small rotations is negligible; therefore, the lateral position and velocity of the body in the laboratory frame can be simplified as

\[
y(x,t) = (x - \pi)\theta + A \sin(x + t) \quad \text{[S2]}
\]

\[
v_y(x,t) = y(x,t) = A \cos(x + t) + \theta(t)(x - \pi).
\]

Using viscous resistive forces, the torque balance equation in Eq. 2 becomes

\[
r_{\text{net}}(t) = 0
= 2\pi
\int \frac{\gamma(y(t),x,\text{CoM})}{x} \, dx = 0
\]

\[
\theta(t) = \frac{3}{\pi} A \cos t
\]

\[
y(x,t) = A \sin(x + t) + \frac{3}{\pi} A \cos t(x - \pi).
\]

**Elastic and Damping Properties of the Sandfish Body**

**Experiment 1: Dynamic Bending Tests.** Torque vs. angle work loops occurred in a clockwise direction for all sandfish (\(n = 3\)) (Fig. 7 and Table S1), indicating energy dissipation. Elasticity changed between the animals tested and increased with speed (Table S2) within a range of \(1 - 20\)°/s. Average elasticity across all animals at \(1\)°/s was 0.12 N cm/rad; at \(10\)°/s, it was 0.17 N cm/rad; and at \(20\)°/s, it was 0.18 N cm/rad. We found torque at zero displacement was constant between 1, 10, and \(20\)°/s (\(P > 0.05\)), leading to a decreasing damping coefficient, \(c\), with increasing speed. For the hysteretic damping model, we found that the structural damping coefficient, \(h\), was independent of angular speed between 1 and \(20\)°/s (\(P > 0.05\)), resulting in a decreasing loss factor, \(\eta\) (\(P < 0.0001\)), due to the increasing body stiffness, \(K\). Trends for estimated torque during swimming using calculated \(K\) and \(c\) are shown in Fig. S4. Torque from hysteretic damping is proportional to torque from viscous damping and follows the same trends.

For the experiment in which a sandfish was rotated at 1, 10, 20, 50, and \(100\)°/s (the maximum achievable value with our system) (Fig. S4, *Inset*), we found similar results for rotation rates below \(20\)°/s. The stiffness coefficient and the associated torque increased from 1 to \(20\)°/s, whereas the torque from both viscous and hysteretic damping remained approximately constant. In accord with this, the area enclosed by the hysteresis loop (\(E_{\text{hyst}}\)) was also constant. However, from 20 to 100°/s, the stiffness coefficient and elastic torque remained constant, whereas both damping torques increased, indicating that viscous damping may be a better model of the system at high angular velocities. The average torque at 240°/s (the average angular velocity of segments during sandswimming) was calculated using the average stiffness and viscous damping coefficient at (20°/s). We estimated that torque due to bending stiffness is 0.09 ± 0.03 N cm, which is ~2.3% of the torque exerted on the sandfish by the external media. The torque due to viscous damping is 0.06 ± 0.034 N cm, and it is 1.5% of total external force. By interpolating the hysteretic damping torque vs. angular speed curve after 20°/s, we estimated similar torque (0.06 N cm) at 240°/s compared with the viscous model.

**Experiment 2: Pendulum Swing Tests.** Fitting a viscoelastic model and hysteretic model (Fig. S6) yielded small spring coefficients (\(K = 0.006 ± 0.1\)) with some calculated values in the negative range (Table S3). We explain this because the torque due to gravity is large compared with the torque from passive elasticity during bending (on average, 7% of the gravitational force), and therefore is not beyond the error of approximation. However, this finding confirms that passive elasticity in sandfish is small.

The viscous damping coefficient (0.012 ± 0.006 N cm·s·rad⁻¹) was on the same order as that measured in the bending test at 20°/s (Table S3). The loss factor also followed the pattern predicted by the bending test. The value of \(\eta\) was ~0.38 ± 0.22 N cm·rad⁻¹. Therefore, the average torque due to hysteretic damping is 0.009 ± 0.004 N cm (less than 0.3% of the torque due to the external load). Regardless of the model chosen, viscous or hysteretic, due to passive damping was small.

In conclusion, both the swing test and the bending test show that passive stiffness and damping are small compared with the force due to the external load (~5% combined) and confirm the assumptions used in our model.

**Estimation of the Torque from Inertial Force for the Sand-swimming of the Sandfish**

The maximal inertial force per unit length can be estimated from the product of the maximal acceleration and body mass per unit length: \(F_{\text{inert}} = (2\pi f)^2 A \times m/L \approx (6.28 \times 2 \text{Hz})^2 \times 1.7 \text{ cm} \times 17.3 \text{ g/14.7 cm} = 0.003 \text{ N/cm}.\) We estimate that the effective density of the glass beads is \(\rho_{\text{eff}} = \rho_{\text{glass}}\phi = 1.5 \text{ g/cm}^3\) and the volume of material that moves with the body is about the same as that of the body itself. Then, the total inertial force is about 0.008 N/cm. Assuming the ratio between the torque from inertial force and resistive force is the same as the ratio between the forces (0.008 N/cm × 0.8 N/cm = 0.01), we estimate the torque from inertial force as 4 N cm × 0.01 = 0.04 N cm.


Fig. S1. Empirical force relationships used in the resistive force theory model [adapted from the study by Maladen et al. (1)]. Green solid lines represent the perpendicular \(F_\perp\) and parallel \(F_\parallel\) components of the force. Dashed gray lines correspond to \(F_\perp\) and \(F_\parallel\) calculated for an infinitely long slender ellipsoid in a low Reynolds number fluid by choosing a velocity that fits \(F_\parallel\) vs. \(\psi\).

Fig. S2. Lateral position of the swimmer in its body frame (A) and in the laboratory frame (B). (A and C) Position of the model swimmer in the body frame is a traveling sinusoidal wave with \(A/\lambda = 0.22\). Color represents lateral displacement \(y_b\). (B and D) Position of a swimmer in a viscous fluid in the laboratory frame. The swimmer uses a sinusoidal wave with a small amplitude (\(A/\lambda = 0.01\) in this example) and the fore-aft distance is kept as \(2\pi\). Color represents lateral displacement \(y\). The analytical expression is provided in Eq. S3.
Fig. S3. The body elasticity is experimentally measured by bending the sandfish body and measuring the resulting torque. (A) Top and (B) side views of the experimental setup are shown. Adjustable grips hold the animal at different snout-to-vent length (SVL) locations (0.5 and 0.6 SVL) and are attached to a rigid platform and to a rotating platform, respectively. A motor rotates the anterior region of the sandfish through ±15°, and resulting bending moment is measured with strain gauges. Black circles were marked on the animal's midline at increments of 0.1 SVL. The best-fit line through the markers circled in red was used to calculate angle θ.
Fig. S4. Estimated average torque using calculated elasticity and damping coefficients shown for animal 1 (blue circles), animal 2 (red circles), and animal 3 (green circles) (n = 8 trials each). (Insets) Trends for the experiment at higher speeds (magenta circles). (A) Torque slightly increases due to a higher $K$ up to $20^\circ$ and then plateaus afterward. (B) Torque due to viscous damping remains constant (due to decreasing $c$) up until $20^\circ$ and increases afterward (due to constant $c$). (C) Area contained within the best-fit curves between the torque and angle is independent of speed before $20^\circ$. 

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Fig. S5. Swing test to measure passive body properties at natural frequencies. Sandfish were clamped with soft foam to restrict movement anterior to 0.5 SVL. The initial position is shown (dashed outline, $t = 0$ s). The angle is calculated after the first half-cycle (dotted outline, $t = 0.33$ s) when the angle relative to 0.5 SVL and bending is smaller. (Inset) Sandfish body is modeled as a cylinder and a cone with uniform density, where $d_{\text{com}}$ is the distance to the center of mass.

Fig. S6. Representative angle vs. time trajectory for animal 1 during a pendulum swing test. The pink line represents the experiment, the blue dashed line is the viscous model, and the green dashed line is the hysteretic model with best-fit parameters.

Table S1. Mass and size of three sandfish used to estimate passive body properties

<table>
<thead>
<tr>
<th>Sandfish</th>
<th>Mass, g</th>
<th>Snout-to-vent length, cm</th>
<th>Body length, cm</th>
<th>Width, cm</th>
<th>Height, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>10.1</td>
<td>15.7</td>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>8.6</td>
<td>14.5</td>
<td>1.5</td>
<td>1.1</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>8.2</td>
<td>13.4</td>
<td>1.5</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Sandfish 2 was used again for the high-speed rotation test.
Table S2. Work loop parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
<th>df</th>
<th>F</th>
<th>P</th>
<th>Animal no.</th>
<th>Mean ± SD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/s</td>
</tr>
<tr>
<td>K, N-cm/rad</td>
<td>Frequency</td>
<td>1</td>
<td>151.7</td>
<td>&lt;0.0001</td>
<td>Sandfish 1</td>
<td>0.175 ± 0.017</td>
</tr>
<tr>
<td></td>
<td>Animal</td>
<td>2</td>
<td>214.62</td>
<td>&lt;0.0001</td>
<td>Sandfish 2</td>
<td>0.084 ± 0.009</td>
</tr>
<tr>
<td></td>
<td>Frequency × animal</td>
<td>2</td>
<td>4.14</td>
<td>0.0203</td>
<td>Sandfish 3</td>
<td>0.088 ± 0.011</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>66</td>
<td></td>
<td></td>
<td>Overall</td>
<td>0.116 ± 0.045</td>
</tr>
<tr>
<td>c, N-cm·rad⁻¹</td>
<td>Frequency</td>
<td>1</td>
<td>82.36</td>
<td>&lt;0.0001</td>
<td>Sandfish 1</td>
<td>0.418 ± 0.207</td>
</tr>
<tr>
<td></td>
<td>Animal</td>
<td>2</td>
<td>5.48</td>
<td>0.0063</td>
<td>Sandfish 2</td>
<td>0.151 ± 0.092</td>
</tr>
<tr>
<td></td>
<td>Frequency × animal</td>
<td>2</td>
<td>5.85</td>
<td>0.0046</td>
<td>Sandfish 3</td>
<td>0.298 ± 0.072</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>66</td>
<td></td>
<td></td>
<td>Overall</td>
<td>0.289 ± 0.172</td>
</tr>
<tr>
<td>η</td>
<td>Frequency</td>
<td>1</td>
<td>21.01</td>
<td>&lt;0.0001</td>
<td>Sandfish 1</td>
<td>0.157 ± 0.0718</td>
</tr>
<tr>
<td></td>
<td>Animal</td>
<td>2</td>
<td>12.65</td>
<td>&lt;0.0001</td>
<td>Sandfish 2</td>
<td>0.123 ± 0.073</td>
</tr>
<tr>
<td></td>
<td>Frequency × animal</td>
<td>2</td>
<td>0.04</td>
<td>0.9593</td>
<td>Sandfish 3</td>
<td>0.231 ± 0.078</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>66</td>
<td></td>
<td></td>
<td>Overall</td>
<td>0.170 ± 0.085</td>
</tr>
<tr>
<td>Eloss, N-cm·rad⁻¹</td>
<td>Frequency</td>
<td>1</td>
<td>0.39</td>
<td>0.5355</td>
<td>Sandfish 1</td>
<td>0.006 ± 0.002</td>
</tr>
<tr>
<td></td>
<td>Animal</td>
<td>2</td>
<td>19.7</td>
<td>&lt;0.0001</td>
<td>Sandfish 2</td>
<td>0.004 ± 0.001</td>
</tr>
<tr>
<td></td>
<td>Frequency × animal</td>
<td>2</td>
<td>2.22</td>
<td>0.1168</td>
<td>Sandfish 3</td>
<td>0.005 ± 0.001</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>66</td>
<td></td>
<td></td>
<td>Overall</td>
<td>0.005 ± 0.002</td>
</tr>
</tbody>
</table>

One-way analysis of covariance table testing effect of oscillation frequency and animal on stiffness, K; viscous damping, c; loss factor, η; and area, Eloss. Average values are shown and separated by speed and animal.

Table S3. Average swing test parameter values (mean ± SD) using viscous and hysteretic damping models for each sandfish and for all data (overall)

<table>
<thead>
<tr>
<th>Animal no.</th>
<th>Osc. freq., s⁻¹</th>
<th>K, N-cm·rad⁻¹</th>
<th>c, N-cm·s·rad⁻¹</th>
<th>η</th>
<th>Osc. freq., s⁻¹</th>
<th>K, N-cm·rad⁻¹</th>
<th>η</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandfish 1</td>
<td>2.21 ± 0.04</td>
<td>−0.115 ± 0.030</td>
<td>0.009 ± 0.002</td>
<td></td>
<td>2.20 ± 0.05</td>
<td>−0.127 ± 0.016</td>
<td>0.183 ± 0.015</td>
</tr>
<tr>
<td>Sandfish 2</td>
<td>2.70 ± 0.08</td>
<td>0.092 ± 0.035</td>
<td>0.020 ± 0.005</td>
<td></td>
<td>2.54 ± 0.02</td>
<td>−0.016 ± 0.007</td>
<td>0.698 ± 0.103</td>
</tr>
<tr>
<td>Sandfish 3</td>
<td>2.73 ± 0.01</td>
<td>0.055 ± 0.037</td>
<td>0.009 ± 0.001</td>
<td></td>
<td>2.69 ± 0.08</td>
<td>0.039 ± 0.029</td>
<td>0.320 ± 0.025</td>
</tr>
<tr>
<td>Overall</td>
<td>2.54 ± 0.26</td>
<td>0.006 ± 0.100</td>
<td>0.012 ± 0.006</td>
<td></td>
<td>2.47 ± 0.23</td>
<td>−0.036 ± 0.075</td>
<td>0.383 ± 0.224</td>
</tr>
</tbody>
</table>

Osc. freq., oscillation frequency.
Movie S1. Muscle activation of a sandfish during sand-swimming from the electromyogram experiment (upper half) and the resistance force theory model (lower half). Subsurface swimming of the sandfish is recorded using high-speed X-ray imaging. Black opaque markers (black circles) are attached to the exterior midline to facilitate tracking. Colored dots indicate muscle activation at 0.5, 0.7, 0.9, and 1.1 SVLs, respectively. The thick gray line also indicates muscle activation in the model. Green arrows represent forces. Magenta arrows represent velocities. The experiment in this movie is slowed by 12.5×.